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Distributed Feedback Control of the Benjamin-Bona-Mahony-Burgers Equation by a Reduced-Order Model

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Abstract. A reduced-order model for distributed feedback control of the Benjamin-Bona-Mahony-Burgers (BBMB) equation is discussed. To retain more information in our model, we first calculate the functional gain in the full-order case, and then invoke the proper orthogonal decomposition (POD) method to design a low-order controller and thereby reduce the order of the model. Numerical experiments demonstrate that a solution of the reduced-order model performs well in comparison with a solution for the full-order description.

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1. Introduction

Standard discretisation schemes (finite element, finite difference, spectral element, finite volume, etc.) may require quite high degree for accurate simulation of fluid flows, and can be expensive with respect to both storage and computing time. Reduced-order models for the simulation of nonlinear complex systems and optimal or feedback control has therefore received more attention recently. This approach involves projecting the dynamical system onto subspaces consisting of basis elements that reflect characteristics of the expected solution, in contrast to the traditional numerical methods — e.g. the elements of the subspaces in the finite element method are uncorrelated to the physical properties of the system described.

The proper orthogonal decomposition (POD) method has also received considerable attention in recent years, as a tool to analyse complex physical systems. This method begins with a set of snapshots generated by either evaluating the computed solution in transient

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problems at several instants of time or by evaluating the computed solution for several values of the parameters appearing in the problem description, or a combination of both. In this article, the snapshots are obtained by a finite element method, from which the POD basis is constructed — viz. the left singular vectors corresponding to the most dominant singular values of the matrix where the columns are the snapshot vectors. This basis is then used to determine an approximate solution for different values of the system parameters, usually by a projection procedure. POD-based model reduction has been applied with some success to several problems [2, 6, 7, 9, 12, 13, 15, 18, 19, 22–25, 27–30, 32, 33].

We propose and test a reduced-order model for a distributed feedback control problem involving the Benjamin-Bona-Mahony-Burgers (BBMB) equation, which describes the propagation of small amplitude long waves in nonlinear dispersive media [5] — viz.

$$\begin{cases} y_t - y_{xxt} - \alpha y_{xx} + \beta y_x + y y_x = f(x,t) \ \text{ in } \Omega \times [0,T]\,, \\ y(0,t) = y(L,t) = 0 \ \text{ on } [0,T]\,, \\ y(x,0) = y_0(x) \ \text{ in } \Omega\,, \end{cases} \tag{1.1}$$

where $\Omega = [0, L]$, $\alpha > 0$ and β are constants and $f(x,t)$ is a given forcing term. The physical dispersion in the BBMB equation is the same as in the Benjamin-Bona-Mahony (BBM) equation and the dissipation is the same as in the Burgers equation, providing an alternative to the Korteweg-de Vries-Burgers (KdVB) equation [17]. Stabilisation of the boundary feedback control for the BBM, KdVB and Burgers equations has been investigated in Refs. [3, 4, 16, 18]. In the case of the KdV equation, the feedback controller is locally and globally controllable and stabilisable. When it is small, the solution of the generalised regularised long wave-Burgers (GRLWB) equation decays like the solution of the corresponding linear equation [8]. Since the BBMB equation is an important case of the GRLWB equation, we can design the controller and control the system (1.1) using a linear-quadratic regulator method. Here the quadratic B-spline finite element method is adopted to convert the BBMB equation into a finite set of nonlinear ordinary differential equations, in designing the full-order control law. Using the reduced-order basis obtained by the POD method, we then design the low-order controller.

In Section 2, we describe the B-spline finite element approximation of solution of the BBMB equation. In Section 3, we briefly review POD-based reduced-order bases, and in Section 4 we discuss our numerical scheme for the distributed feedback control problem. Some numerical results are given in Section 5, followed by a brief Conclusion in Section 6.

2. Finite Element Approximation

2.1. Formulation

Finite element methods (FEM) have often been applied to solve various linear and nonlinear partial differential equations (PDE). Standard Lagrangian finite element basis functions provide only simple C^0-continuity, and therefore cannot be used for the spatial discretisation of higher-order (e.g. third-order or fourth-order) differential equations. On the