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Response and Recovery Times of Elastic and Viscoelastic Capsules in Shear Flow

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Abstract. Amid the recent interest in the role of membrane viscosity in the deformation of a fluid-filled capsule, we consider the role of various capsule properties (shear elasticity, membrane bending stiffness and viscosity) in determining the response and recovery times of a spherical capsule in shear flow. These times are determined by fitting exponential functions to results for the Taylor deformation parameter D_{xy}. We focus on the relationship between the membrane and fluid viscosity ratios, as suggested by Diaz *et al* [8], and whether adjustments to the fluid viscosity ratio may be used to approximate the effects of membrane viscosity. Based on its ability to reproduce response and recovery times, our results suggest that such an approach holds promise.

AMS subject classifications: 74F10

Key words: Elastic capsule, viscoelastic capsule, response time, recovery time, shear flow.

1 Intro

Diaz and coworkers [9] considered the response and recovery of an elastic capsule in elongational flow. Finding that the capsule's response and recovery times could be determined by an exponential fitting, they investigated the role of parameters such as the fluid viscosity ratio and the capillary number. The response and recovery times of the capsule are important to understanding how a capsule will react in more complicated, time-dependent flows, as arise in medical and industrial applications [10]. Diaz *et al* [8] extended their consideration to a viscoelastic capsule in elongational flow and compared the respective impacts of different fluid and membrane viscosity ratios.

However, many of the attractive aspects of elongational flow, such as simpler computation due to axisymmetry, may also potentially restrict the applicability of its results.

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For instance, fluid and membrane viscosity ratios do not affect steady-state shape of the capsule in elongational flow, and the capsule's membrane does not exhibit any tank-treading [2]. In contrast, the deformation of a capsule in shear flow has dynamical and angular aspects which do not occur in elongational flow. The steady-state shape of a capsule in shear flow depends on a range of parameters - capillary number, bending stiffness, membrane and fluid viscosity ratios – as do the capsule's angle of inclination and tank-treading frequency.

Nonetheless, Diaz et al [9] applied their exponential-fitting methodology to the results of Ramanujan and Pozrikidis [23] for the deformation of a spherical capsule in shear flow. They found instructive parallels between their results, including a near constant ratio between response times in elongational and shear flows, for capsules with equal steady-state deformation, as measured by the Taylor deformation parameter. This striking comparison, however, is limited because Ramanujan and Pozrikidis did not model the membrane viscosity, and neither study incorporated bending stiffness. We aim to consider this analogy in a more complete setting, with viscoelastic capsules that may resist bending and have a non-unity fluid viscosity ratio.

We consider the deformation response of a spherical capsule in shear flow, along with its shape recovery after the shear flow is abruptly stopped. Our methodology uses a finite element structural method and treats the fluid with a lattice Boltzmann method, coupling the structure and fluid with the immersed boundary method. The structural model considers the capsule's elasticity, membrane viscosity, and bending stiffness, while the fluid model permits different fluid viscosities inside and outside of the capsule. We quantify the shape change of the capsule using the Taylor deformation parameter and fit an exponential curve to this parameter to determine the response and recovery times.

2 Algorithms

2.1 Fluid

The incompressible Navier-Stokes equations are solved using a lattice Boltzmann method (LBM). Derived from the Boltzmann equation of statistical mechanics, the lattice Boltzmann method considers the fluid to be sets of particles that move between lattice nodes in discrete timesteps with discrete velocities. Despite its statistical origins, the lattice Boltzmann method is deterministic, using the averaged behaviour of particles.

The expression $\mathbf{f}_i(\mathbf{x}_j,t_n)$ represents the distribution of particles at $\mathbf{x}_j$ with velocity $\mathbf{c}_i$ at time t_n. The discrete velocities $\mathbf{c}$ are from the D3Q19 lattice model and we set $h=dx=dt$. Using a multiple relaxation time (MRT) approximation of the collision integral, we have the lattice Boltzmann equation

$$\mathbf{f}(\mathbf{x}_j+\mathbf{c}dt,t_n+dt)-\mathbf{f}(\mathbf{x}_j,t_n)=-\mathbf{M}^{-1}\mathbf{S}\left[\mathbf{m}(\mathbf{x}_j,t_n)-\mathbf{m}^{(\mathrm{eq})}(\mathbf{x}_j,t_n)\right] \tag{2.1}$$

in which we denote the probability distribution functions by $\mathbf{f}$, their velocity moments by $\mathbf{m}$, and their equilibrium moments by $\mathbf{m}^{(eq)}$ [6,7,13].