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A Slope Constrained 4th Order Multi-Moment Finite Volume Method with WENO Limiter

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Abstract. This paper presents a new and better suited formulation to implement the limiting projection to high-order schemes that make use of high-order local reconstructions for hyperbolic conservation laws. The scheme, so-called MCV-WENO4 (multi-moment Constrained finite Volume with WENO limiter of 4th order) method, is an extension of the MCV method of Ii & Xiao (2009) by adding the 1st order derivative (gradient or slope) at the cell center as an additional constraint for the cell-wise local reconstruction. The gradient is computed from a limiting projection using the WENO (weighted essentially non-oscillatory) reconstruction that is built from the nodal values at 5 solution points within 3 neighboring cells. Different from other existing methods where only the cell-average value is used in the WENO reconstruction, the present method takes account of the solution structure within each mesh cell, and thus minimizes the stencil for reconstruction. The resulting scheme has 4th-order accuracy and is of significant advantage in algorithmic simplicity and computational efficiency. Numerical results of one and two dimensional benchmark tests for scalar and Euler conservation laws are shown to verify the accuracy and oscillation-less property of the scheme.

AMS subject classifications: 65M08, 65M70, 76L05, 76N15

Key words: Multi-moment finite volume method, WENO, flux reconstruction, compressible flow, conservative method, oscillation-suppressing.

1 Introduction

High order numerical methods have got increasingly use in solving fluid dynamic problems for their superior performance in resolving the vortex-dominant flows in comparison with low order methods. Different from the conventional finite volume method

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(FVM) and finite difference method (FDM), making use of locally (cell-wisely) increased degrees of freedoms (DOFs) to construct high-order schemes is a trend for the past decades in the field of computational fluid dynamics (CFD), which still remains an active research direction. The major advantages of a scheme using high-order local reconstruction (HLR) lie in the spectral-like convergence rate and the adaptivity to unstructured grids. Some representative methods of this sort for CFD applications are the spectral element method [19], the discontinuous Galerkin (DG) method [4–6, 9], the constrained interpolation profile (CIP) method [45, 46], the staggered-grid (SG) Chebyshev multidomain method [16], the spectral volume (SV) method [34, 35], the spectral difference (SD) method [30], multi-moment (constrained) finite volume (MV or MCV) method [3, 13, 14, 43, 44].

Although apparent differences are seen among the aforementioned methods in the details of reconstruction and solution procedures, all of them realize high-order accuracy via locally reconstructed polynomials and guarantee the numerical conservation by introducing an FVM-like constraint condition on the continuity of numerical fluxes across mesh cell boundaries, which are computed through exact or approximate Riemann solvers. From this observation, a general framework for constructing high-order spectral-convergent schemes, so-called Flux Reconstruction (FR) method, was proposed in [12]. The FR formulation treats the point values as the computational variable at the solution points located within each grid cell, which facilitates local reconstructions of high order. An FR scheme computes the point-wise solution via the differential form of the governing equations where the flux function must be reconstructed so as to satisfy the continuity on the cell boundaries. As shown in [12], nearly all existing nodal-value based high order schemes can be interpreted as the subset cases of the FR framework where the correction functions involved in the flux reconstruction procedure makes the difference among the schemes. The FR was extended to unstructured grids under the name of CPR (correction procedure via reconstruction) [36].

We show in the multi-moment constrained flux reconstruction (MMC-FR) method [38] that the flux function can be reconstructed more flexibly with a wider variety of constraint conditions. We demonstrated that stable and more efficient schemes can be devised by making use of not only the point values but also the derivatives of different orders at different constraint points. The class of MCV schemes [14] can be devised straightforwardly from various constraint conditions under the MMC-FR framework.

Despite the superiority of the schemes with HLR in resolving complex structures of smoothness, numerical oscillations associated with discontinuities turns out to be another problem. Using some special interpolation function, such as the rational function in [39, 40, 42], proves to be effective in suppressing numerical oscillations. Being a more general approach, nonlinear limiting projection can be used to prevent the spurious oscillations in the presence of discontinuities or large jumps. For example, total variation bounded (TVB) limiters were used in the DG method [4–6] and other HLR methods mentioned before, and proved to be effective in computing even shock-dominant flows. Compared to the TVB limiter, the weighted essentially non-oscillatory (WENO) limiter [15, 18] is more attractive because it is able to effectively suppress the numerical oscillation in