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Non-Relativistic and Low Mach Number Limits of a Compressible Full MHD-$P1$ Approximate Model Arising in Radiation Magnetohydrodynamics

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Abstract. In this paper we study the non-relativistic and low Mach number limits of strong solutions to a full compressible MHD-$P1$ approximate model arising in radiation magnetohydrodynamics. We prove that, as the parameters go to zero, the solutions of the primitive system converge to that of the classical incompressible magnetohydrodynamic equations.

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Key words: Full MHD-$P1$ approximate model, non-relativistic and low Mach number limit.

1 Introduction

In this paper we consider the following full compressible MHD-$P1$ approximate model arising in radiation magnetohydrodynamics [2,4,7]:

$$\partial_t\rho+\operatorname{div}(\rho u)=0, \tag{1.1a}$$

$$\partial_t(\rho u)+\operatorname{div}(\rho u\otimes u)+\frac{1}{\epsilon_1^2}\nabla p-\mu\Delta u-(\lambda+\mu)\nabla\operatorname{div}u=I_1+\operatorname{rot}b\times b, \tag{1.1b}$$

$$\begin{aligned}&\partial_t(\rho e)+\operatorname{div}(\rho ue)+p\operatorname{div}u-\Delta\mathcal{T}\\&\qquad=\epsilon_1^2\Big(\frac{\mu}{2}|\nabla u+\nabla u^t|^2+\lambda(\operatorname{div}u)^2+|\operatorname{rot}b|^2\Big)+I_0-\mathcal{T}^4,\end{aligned} \tag{1.1c}$$

$$\partial_t b+\operatorname{rot}(b\times u)-\Delta b=0,\quad \operatorname{div}b=0, \tag{1.1d}$$

$$\epsilon_2\partial_t I_0+\operatorname{div}I_1=\mathcal{T}^4-I_0, \tag{1.1e}$$

$$\epsilon_2\partial_t I_1+\nabla I_0=-I_1\ \text{ in }\ \mathbb{T}^3\times(0,\infty), \tag{1.1f}$$

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where $\rho,u,\mathcal{T},b$ and $I:=I_0+I_1\cdot\omega$ denote the density, velocity, temperature, magnetic field, and the radiation intensity of the fluid, respectively. $\omega\in\mathbb{S}^2$ is the direction vector. The viscosity coefficients μ and λ of the fluid satisfy $\mu>0$ and $\lambda+\frac{2}{3}\mu\geq 0$. $\epsilon_1>0$ is the (scaled) Mach number and $\epsilon_2>0$ is the (scaled) light speed. $\mathbb{T}^3$ is a periodic domain in $\mathbb{R}^3$.

When the magnetic field in (1.1) is neglected, i.e. $b=0$, the system is reduced to the Navier-Stokes-Fourier-$P1$ model, and the papers [4,8,14] established non-relativistic and low Mach number limits of the problem.

If we ignore the radiation effort in (1.1), the system is reduced to full compressible magnetohydrodynamic equations and has received many studies, for example, see [3,6, 9–13]. The local strong solution was obtained by Fan-Yu [9]. The global weak solutions were obtained by Fan-Yu [10], Ducomet-Feireisl [6] and Hu-Wang [11] respectively. The low Mach number limit problems were studied by Jiang-Ju-Li [12] in $\mathbb{T}^3$ for well-prepared initial data, Jiang-Ju-Li-Xin [13] in $\mathbb{R}^3$ for ill-prepared initial data, and Cui-Ou-Ren [3] in a bounded domain for well-prepared initial data.

Very recently, Xie and Klingenberg [16] studied the non-relativistic limit for the three-dimensional ideal compressible radiation magnetohydrodynamics and obtained the limiting problem which is a widely used macroscopic model in radiation magnetohydrodynamics.

In this paper, we study the non-relativistic and low Mach number limits to the full compressible MHD-$P1$ approximate model (1.1) and hence extend the result in [4] to more general models. For simplicity, we will consider the case that the fluid is a polytropic ideal gas, i.e., the internal energy e and the the pressure p satisfy

$$e:=C_V\mathcal{T},\qquad p:=R\rho\mathcal{T}$$

with positive constants C_V and R.

In the following, we introduce the new unknowns σ and θ with

$$\rho:=1+\epsilon_1\sigma,\quad \mathcal{T}:=1+\epsilon_1\theta. \tag{1.2}$$

Then the system (1.1) can be rewritten as

$$\partial_t\sigma+\operatorname{div}(\sigma u)+\frac{1}{\epsilon_1}\operatorname{div}u=0, \tag{1.3a}$$

$$\begin{aligned}&\rho\partial_t u+\rho u\cdot\nabla u+\frac{R}{\epsilon_1}(\nabla\sigma+\nabla\theta)+R\nabla(\sigma\theta)\\&\qquad-\mu\Delta u-(\lambda+\mu)\nabla\operatorname{div}u=I_1+\operatorname{rot}b\times b,\end{aligned} \tag{1.3b}$$

$$\begin{aligned}&C_V\rho(\partial_t\theta+u\cdot\nabla\theta)+R(\rho\theta+\sigma)\operatorname{div}u+\frac{R}{\epsilon_1}\operatorname{div}u-\Delta\theta\\&\qquad=\epsilon_1\Big(\frac{\mu}{2}|\nabla u+\nabla u^t|^2+\lambda(\operatorname{div}u)^2+|\operatorname{rot}b|^2\Big)+I_0-(1+\epsilon_1\theta)^4,\end{aligned} \tag{1.3c}$$

$$\partial_t b+\operatorname{rot}(b\times u)-\Delta b=0,\quad \operatorname{div}b=0, \tag{1.3d}$$

$$\epsilon_2\partial_t I_0+\operatorname{div}I_1=(1+\epsilon_1\theta)^4-I_0, \tag{1.3e}$$

$$\epsilon_2\partial_t I_1+\nabla I_0=-I_1\quad\text{in }\ \mathbb{T}^3\times(0,\infty). \tag{1.3f}$$