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Strong Convergence Analysis of Split-Step θ-Scheme for Nonlinear Stochastic Differential Equations with Jumps

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Abstract. In this paper, we investigate the mean-square convergence of the split-step θ-scheme for nonlinear stochastic differential equations with jumps. Under some standard assumptions, we rigorously prove that the strong rate of convergence of the split-step θ-scheme in strong sense is one half. Some numerical experiments are carried out to assert our theoretical result.

AMS subject classifications: 65C20, 60H35, 60H10

Key words: Split-step scheme, strong convergence, stochastic differential equation, jump-diffusion, one-side Lipschitz condition.

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1 Introduction

We consider *jump-diffusion Itô stochastic differential equations* (JSDEs) of the form

$$\begin{cases} dX(t)=f(X(t_-))dt+g(X(t_-))dW(t)+h(X(t_-))dN(t), & t\in(0,T], \\ X(0_-)=X_0, \end{cases} \tag{1.1}$$

where $X(t_-):=\lim_{s\to t-}X(s)$, $f:\mathbb{R}^m\to\mathbb{R}^m$, $g:\mathbb{R}^m\to\mathbb{R}^{m\times d}$ and $h:\mathbb{R}^m\to\mathbb{R}^m$, $m,d\in\mathbb{N}^+$. Here $W(t)$ is a standard d-dimensional Brownian motion, and $N(t)$ is a scalar Poisson process (independent of $W(t)$) with intensity $\lambda>0$, both defined on a complete probability space $(\Omega,\mathcal{F},\mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., it is increasing and right continuous and $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). Extension of our work to vector-valued jumps with independent entries is straightforward.

Stochastic differential equations (SDEs) have been widely used in many areas such as chemistry, physics, engineering, biology and mathematical finance to provide models of

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dynamical systems affected by uncertainty factors. When it is the case that a stochastic system is also influenced by some randomly occurring impulses it is often desirable to use a jump-diffusion stochastic model such as (1.1) to characterize these burst phenomena. For more practical applications, one can refer to [1,4,5,7,24].

Since dynamical systems modeled by SDEs rarely admit known explicit solutions, seeking accurate numerical solutions has become a rapidly growing research area. In recent years, much progress has been made in developing numerical methods for solving SDEs [6,8,9,13–15,17,21,22,25]. However, compared with the development of numerical methods for SDEs, numerical methods for solving JSDEs are far from undeveloped, and thus effective and efficient numerical methods are urgently needed. In addition, most of the existing numerical methods for (1.1) are based on globally Lipschitz conditions and linear growth conditions (see, e.g., [1,2,11,18,19,23]) on the coefficients f, g and h. However, these conditions may be too restrictive, which may exclude lots of useful models to be considered, such as some nonlinear problems with super-linearly growing condition coefficients. To relax the conditions, a popular choice is to use one-sided Lipschitz condition on the drift coefficient and globally Lipschitz conditions on the diffusion and jump coefficients [10,12]. Motivated by the above discussions, we aim to design solvers for (1.1) with weaker conditions on the coefficients f, g and h. More precisely, we will theoretically prove that the split-step θ-scheme (see Section 3), admits a one half rate of strong convergence, under the conditions that the drift coefficient f satisfies one-sided Lipschitz condition and the diffusion coefficient g and jump coefficient h satisfy the globally Lipschitz condition.

The rest of this paper is organized as follows. In Section 2, we introduce notations and assumptions. The split-step θ-scheme is introduced in Section 3. In Section 4, we rigorously obtain the boundedness of the solutions of (1.1) and (3.1a). The boundedness will play a key role in our proof of the convergence error estimates of the split-step θ-scheme. Strong convergence estimates are established in Section 5. In Section 6 we present numerical results to validate our theoretical findings. Finally some conclusions are given in Section 7.

2 Notations and assumptions

Throughout the paper, $\langle\cdot,\cdot\rangle$ denotes the scalar inner product in $\mathbb{R}^m$ or $\mathbb{R}^{m\times d}$, and $|\cdot|$ is the associated Euclidean vector norm or Frobenius matrix norm.

We assume the drift coefficient f satisfies the local Lipschitz condition, i.e., for each $R>0$,

$$|f(x)-f(y)|^2\le L_R|x-y|^2 \tag{2.1}$$

for all $x,y\in\mathbb{R}^m$ with $|x|\vee|y|\le R$, and the one-side Lipschitz condition

$$\langle x-y,f(x)-f(y)\rangle\le K_1|x-y|^2 \quad \text{for all } \ x,y\in\mathbb{R}^m, \tag{2.2}$$