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# A Two-Constraint Method for Appropriate Determination of the Configuration of Source and Collocation Points in the Method of Fundamental Solutions for 2D Laplace Equation

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**Abstract.** Proper positioning of collocation and source points is one of the major issues in the development of the method of fundamental solutions (MFS). In this paper, two constraints for appropriate determination of the location of collocation and source points in the MFS for two-dimensional problems are introduced. The first constraint is introduced to make sure that the solution of the problem has no oscillation between two adjacent collocation points on the boundary. Imposing the second constraint improves the condition of the generated system of equations. In other words, the second constraint reduces the condition number of the MFS system of equations. In this method, no optimization procedure is carried out. The proposed method is formulated for the Laplace problem; however, it can be developed for other problems as well. The accuracy and effectiveness of the proposed method is demonstrated by presenting several numerical examples. It is shown that boundary conditions with a sharp variation of the field variable can be well handled by the presented method. Moreover, it has been found that problems with a concave or re-entrant corner can be efficiently modelled by the proposed two-constraint method.

**AMS subject classifications**: 35A08, 65N80, 65K05, 15A12

**Key words**: Method of fundamental solutions, location of source points, location of collocation points, location parameter, condition number.

## 1 Introduction

The method of fundamental solutions (MFS) is a simple boundary mesh-free method, which has attracted the attention of scientists and engineers, especially over the last two decades. The MFS has been used for the analysis of various kinds of direct and inverse

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problems; see for example [1–4]. The MFS has a potential to give accurate solutions with a very high convergence rate [5,6]. Since the MFS is a boundary mesh-free method, it can be efficiently used for analysis of problems with an unknown or moving boundary [7–9]. Analogous to the boundary element method (BEM), the MFS is based on the knowledge of a fundamental solution of the problem. However, unlike the BEM, the MFS is an integration-free method and this is the advantage of the MFS over the BEM. The evaluation of nearly singular [10, 11] or singular integrals [12, 13] in the BEM requires special techniques, which make the BEM more complicated in comparison with the MFS. Since the MFS uses fundamental solutions as interpolating basis functions, the governing equations are exactly satisfied in the domain and on its boundary by the solution obtained from the MFS. Therefore, the MFS has the potential to provide very accurate numerical solutions simply by satisfying the boundary conditions as accurate as possible. For an accurate satisfaction of boundary conditions in the MFS, one needs to appropriately locate the collocation points on the physical boundary and source points on a pseudo boundary outside the problem domain. Proper positioning of collocation and source points is still one of the major issues in the development of the MFS.

Analysis of a problem via the MFS can be viewed as being equivalent to dealing with an inverse source problem. When the distance from the source points to the collocation points is much larger than the distance between the sources, the problem becomes an ill-posed inverse source problem [14], which results in a corresponding ill-conditioned system of equations. It is well known that a special treatment is required for the stable solution of ill-conditioned system of equations. The Tikhonov regularization method [15, 16], the singular value decomposition (SVD) [17, 18], the damped SVD [19], and smoothing methods [20] can be used for the treatment of ill-conditioned system of equations. Iterative methods such as the conjugate gradient method [21] can also be used for solving system of linear equations. Iterative methods are suitable for large scale problems and can be used for parallel processing more effectively; however, they need some modifications and preconditioning techniques for solving ill-conditioned system of equations [22, 23].

In recent years, some researchers have investigated the determination of the location of the source points in the MFS. Tsai et al. [24] presented a method for locating the sources in the MFS. They have stated that the best accuracy can be obtained when the sources are located far from the boundary and the condition number approaches the limit of the equation solver. They examined their method for different time-independent operators over several simple domains with smooth boundary conditions. They have suggested the nonlinear optimization [25] or domain decomposition method [26] for problems with more complicated domains.

Gorzelaczyk and Koodziej [27] used the MFS to solve several torsion problems with different configurations of source points. They concluded that the error of the MFS is lower if the source points are located on a boundary geometrically similar to the physical boundary than when the source points are located on a circle enclosing the domain of interest. If the boundary shape and boundary conditions are both smooth, then the MFS converges rapidly. In these cases, a circle of relatively large radius can be considered as