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# Image Denoising via Residual Kurtosis Minimization

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**Abstract.** A new algorithm for the removal of additive uncorrelated Gaussian noise from a digital image is presented. The algorithm is based on a data driven methodology for the adaptive thresholding of wavelet coefficients. This methodology is derived from higher order statistics of the residual image, and requires no a priori estimate of the level of noise contamination of an image.

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**Key words**: Denoising, noise estimation, kurtosis minimization, wavelet shrinkage.

## 1. Introduction

With the mass production of digital images of many kinds, the need for efficient methods to restore corrupted images has become immediate. A major source of image contamination is noise. Image noise may arise from quantization of the image data, transmission errors, electronic interference from the imaging hardware, as well as from other sources [1].

Many methods have been proposed for denoising of digital images. Some are based on finite-impulse response (FIR) filtering, where the noisy image is convolved with a smoothing kernel to reduce the visible effect of noise. This may introduce undesirable artificial blur to an image [16]. Some methods cannot reliably distinguish between noise and edge information in an image; they may require a priori knowledge of the noise level in the contaminated image, or might introduce waveform artifacts. A recent discussion of many available methods can be found in [2].

Denoising methods based on the discrete wavelet transform are popular. These methods are similar to spectral filtering methods, but can take advantage of the partial

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localization in space and frequency offered by wavelet transforms. This allows efficient separation of noise from impulsive signal information, such as edges [10, 11]. Denoising is achieved by reducing the magnitude of specific wavelet coefficients.

Many denoising methods assume that the contaminating noise has a particular distribution with known parametrization. That is, some a priori estimate of the noise parameters must be available before denoising can be performed. This paper proposes a new wavelet-based method for the reduction of Gaussian noise in a digital image, and does not require an estimate of the noise parameters to be available. Instead, the method uses the kurtosis statistic of the residual image formed as the difference between the available noisy image and the computed candidate denoised image to determine how much denoising should be carried out. Our approach is based on the observation that the kurtosis statistic is minimal when the residual image is made up of Gaussian noise. We determine the amount of denoising to be carried out by minimizing the kurtosis statistic of the residual image. When the kurtosis statistic is minimal, the residual image is likely to be made up of Gaussian noise, i.e., to be the noise contamination of the available image. We remove the noise in the contaminated image by reducing the magnitude of specific wavelet coefficients in its representation (soft shrinkage). This approach makes it possible to dispense with an a priori estimate of the noise variance in the available noise-contaminated image.

A few numerical examples show our denoising method to be competitive with the BayesShrink method [3] and a Total Variation (TV) norm based denoising scheme. A careful comparison with the many available denoising methods is outside the scope of the present paper. However, we would like to point out that the kurtosis statistic also can be applied in conjunction with other denoising methods to reduce the number of parameters that have to be chosen by a user. We are currently investigating this approach.

This paper is organized as follows. Section 2 reviews the methodology of denoising through thresholding of wavelet coefficients. In Section 3, we derive our new method, which is based on the use of higher order statistics. Section 4 presents numerical experiments of the denoising of digital images (corrupted by Gaussian noise) using the proposed method. Concluding remarks can be found in Section 5.

## 2. Denoising in the wavelet transform domain

The use of the discrete wavelet transform for filtering additive zero-mean Gaussian noise was first proposed by Donoho and Johnson [10]. Consider the following model for the corruption of a gray-scale digital image by additive noise

$$G = F + E,$$

where $G, F, E \in \mathbb{R}^{m\times n}$. Here $F$ is a matrix that encodes the unavailable noise-free image, the matrix $E$ represents zero-mean, independent and identically distributed