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A Conformal Energy-Conserved Method for Maxwell's Equations with Perfectly Matched Layers

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Abstract. In this paper, a conformal energy-conserved scheme is proposed for solving the Maxwell's equations with the perfectly matched layer. The equations are split as a Hamiltonian system and a dissipative system, respectively. The Hamiltonian system is solved by an energy-conserved method and the dissipative system is integrated exactly. With the aid of the Strang splitting, a fully-discretized scheme is obtained. The resulting scheme can preserve the five discrete conformal energy conservation laws and the discrete conformal symplectic conservation law. Based on the energy method, an optimal error estimate of the scheme is established in discrete L^2-norm. Some numerical experiments are addressed to verify our theoretical analysis.

AMS subject classifications: 65M12, 65M15, 65M70

Key words: Maxwell's equations, Fourier pseudo-spectral method, error estimate, conformal conservation law, PML.

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1 Introduction

Since the initial work of Yee [38], numerical methods have been widely used in solving electromagnetic problems. However, one of the inconveniences of the numerical methods lies in the fact that the Maxwell's equations have to be solved in a bounded domain. Thus, in order to absorb the outgoing waves, one needs to apply some special conditions on the boundaries of the computational domain. In Refs. [2,3], Bérenger firstly proposed the perfectly matched layer (PML) technique, which was based on the use of an absorbing layer especially designed to absorb the electromagnetic waves without reflection from the vacuum-layer interfaces. The basic idea of the PML technique of Bérenger was based on modifying the Maxwell's equations in the absorbing layer. The resulting equations

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are commonly referred to as the Maxwell's equations with the PML [19, 20]. Due to the simplicity, the versatility, and the robust treatment of corners in the practical applications, devising efficient numerical methods for the Maxwell's equations with the PML attracts a lot of interest.

It is well-known that structure-preserving methods or geometric numerical methods have exhibited significant superiority over traditional methods in solving Hamiltonian ordinary differential equations (ODEs) and Hamiltonian partial differential equations (PDEs) (e.g., see Refs. [5, 13, 17, 37] and references therein). In the past few decades, various structure-preserving schemes have been developed for the Maxwell's equations. In Refs. [8, 18, 23, 33, 36, 39], symplectic and multi-symplectic schemes of the Maxwell's equations in an isotropic, lossless and sourceless medium were proposed. In Ref. [11], Chen et al. proposed an energy-conserved splitting method for two dimensional (2D) Maxwell's equations in the isotropic, lossless and sourceless medium. Further analysis in three dimensional (3D) case was investigated in Ref. [12]. Other works on the energy-conserved methods for the Maxwell's equations in the isotropic, lossless and sourceless medium can be found in Refs. [6, 7, 22].

However, since the system of the Maxwell's equations with the PML is neither a conservative system nor a Hamiltonian system, the energy-conserved methods, which were developed for the Maxwell's equations, will lose their advantages when applied directly to the Maxwell's equations with the PML. Thus, designing the numerical schemes for the Maxwell's equations with the PML is challenging. In Ref. [20], by virtue of the splitting technique, Hong, Ji and Kong proposed an energy-dissipation splitting finite-difference time-domain (FDTD) method for the 2D Maxwell's equations with the PML. Subsequently, Hong and Ji [19] studied the energy evolution of multi-symplectic methods for the 3D case. In Ref. [32], Birkhoffian multi-symplectic methods for the Maxwell's equations with the PML were investigated by Su and Li. However, most of existing methods have low order accuracy in space and the rigorous error estimate is not established well.

Recently, there has been growing interest in conformal methods for Hamiltonian systems with a linear damping term (e.g., see Refs. [14, 27, 29]). The conformal method provided clear advantages in preserving the conformal conservation laws and long time simulations over standard methods [4, 29]. Other works most related to the conformal method can be found in Refs. [24, 34, 35]. However, there has been no reference considering a conformal Fourier pseudo-spectral scheme for the Maxwell's equations with the PML to the best of our knowledge. It is shown that Fourier pseudo-spectral methods with a high order accuracy have exhibited obvious superiority over the conventional finite difference method in simulating electromagnetic waves [25] and played an important role in keeping the physical properties of primitive problems (e.g., see Refs. [10, 15, 30, 37] and references therein). Thus, our main attention is focusing on the following two aspects:

1. We propose a novel conformal Fourier pseudo-spectral scheme (CFPS) for the Maxwell's equations with the PML. We show that the proposed scheme can pre-