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A FIRST-ORDER SPLITTING METHOD FOR SOLVING A LARGE-SCALE COMPOSITE CONVEX OPTIMIZATION PROBLEM*

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Abstract

In this paper, we construct several efficient first-order splitting algorithms for solving a multi-block composite convex optimization problem. The objective function includes a smooth function with a Lipschitz continuous gradient, a proximable convex function that may be nonsmooth, and a finite sum composed of a proximable function and a bounded linear operator. To solve such an optimization problem, we transform it into the sum of three convex functions by defining an appropriate inner product space. Based on the dual forward-backward splitting algorithm and the primal-dual forward-backward splitting algorithm, we develop several iterative algorithms that involve only computing the gradient of the differentiable function and proximity operators of related convex functions. These iterative algorithms are matrix-inversion-free and completely splitting algorithms. Finally, we employ the proposed iterative algorithms to solve a regularized general prior image constrained compressed sensing model that is derived from computed tomography image reconstruction. Numerical results show that the proposed iterative algorithms outperform the compared algorithms including the alternating direction method of multipliers, the splitting primal-dual proximity algorithm, and the preconditioned splitting primal-dual proximity algorithm.

Mathematics subject classification: 90C25, 65K10.
Key words: Forward-backward splitting method, Primal-dual, Dual, Proximity operator.

1. Introduction

Let H be a real Hilbert space. Let m be an integer such that $m \geq 1$. For each $i \in \{1, 2, \cdots, m\}$, let G_i be a real Hilbert space. The set of all proper, lower semicontinuous convex functions $f : H \to (-\infty, +\infty]$ is denoted by $\Gamma_0(H)$. In this paper, we consider solving a composite convex optimization problem that takes the form

$$\min_{x \in H} \ f(x) + g(x) + \sum_{i=1}^{m} h_i(B_i x), \tag{1.1}$$

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where $g \in \Gamma_0(H)$ maybe nonsmooth, $f : H \to R$ is a convex differentiable function with an L-Lipschitz continuous gradient for some constant $L \in (0, +\infty)$, for each $i \in \{1, 2, \cdots, m\}$, $h_i \in \Gamma_0(G_i)$ and $B_i : H \to G_i$ is a bounded linear operator. In the following, we always assume that the proximity operators with respect to g and $\{h_i\}_{i=1}^m$ have a closed-form solution. The optimization model (1.1) includes a large number of existing models as special cases. For example,

(i) Let $m = 1$. For simplicity and brevity, we drop the subscript "1." Then, the optimization problem (1.1) reduces to

$$\min_{x \in H} \; f(x) + g(x) + h(Bx), \tag{1.2}$$

which has been studied in [1–5].

(ii) Let $f(x) = 0$. Then, the optimization problem (1.1) becomes

$$\min_{x \in H} \; g(x) + \sum_{i=1}^{m} h_i(B_i x), \tag{1.3}$$

which has been studied in [6, 7].

(iii) Let $g(x) = 0$. Then, the optimization problem (1.1) is equivalent to

$$\min_{x \in H} \; f(x) + \sum_{i=1}^{m} h_i(B_i x), \tag{1.4}$$

which has been studied in [8]. Further, let $B_i = I$ for each $i \in \{1, 2, \cdots, m\}$, where I denotes the identity operator. Then, the optimization problem (1.4) reduces to

$$\min_{x \in H} \; f(x) + \sum_{i=1}^{m} h_i(x), \tag{1.5}$$

which has been studied in [9, 10].

(iv) Let $f(x) = 0$ and $g(x) = 0$. Then, the optimization problem (1.1) reduces to

$$\min_{x \in H} \; \sum_{i=1}^{m} h_i(B_i x), \tag{1.6}$$

which has been studied in [11].

Owing to the emergence of the compressive sensing theory, the problem of minimizing the sum of two convex functions when $f = 0$ or $g = 0$ in (1.2) has attracted considerable attention in recent years. A number of efficient iterative algorithms have been developed to solve such this problem, which has wide application in signal and image processing. Examples include the iterative shrinkage-thresholding algorithm (ISTA) and fast ISTA (FISTA) [12, 13], two-step ISTA [14], primal-dual hybrid gradient (PDHG) algorithm [15], fixed-point continuation (FPC) algorithm [16], primal-dual proximity algorithm (PDPA) [17–19] and primal-dual fixed point algorithm based on proximity operator (PDFP^{2}O) [20, 21].

Operator splitting is the most powerful method for solving monotone inclusion problems, and it can be easily applied to the above-mentioned convex optimization problems. Operator splitting methods include forward-backward splitting [22–24], Douglas-Rachford splitting [25, 26] and forward-backward-forward splitting [27]. As operator splitting methods mainly focus on solving inclusion problems of the sum of two monotone operators (see Definition 2.1), they