Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **11**, No. 5, pp. 1048-1063

DOI: 10.4208/aamm.OA-2018-0236
October 2019

An Adaptive Finite Element Solver for Demagnetization Field Calculation

Lei Yang[1] and Guanghui Hu[2,3,*]

[1] *Faculty of Information Technology, Macau University of Science and Technology, Macao SAR, China*
[2] *Department of Mathematics, Faculty of Science and Technology, University of Macau, Macao SAR, China*
[3] *Zhuhai UM Science & Technology Research Institute, Zhuhai, Guangdong, China*

Received 1 November 2018; Accepted (in revised version) 19 January 2019

Abstract. Quality calculation of the demagnetization field plays an important role in the computational micromagnetics. It is a nontrivial challenge to develop a robust and efficient algorithm to handle the requirements from the practical simulations, since the nonlocality of the demagnetization field evaluation and the irregularity of the computational domain. In [C. J. Garcia-Cervera and A. M. Roma, Adaptive mesh refinement for micromagnetics simulations, IEEE Trans. Magn., 42(6) (2006), PP. 1648–1654], the evaluation of the demagnetization field is split into solving two partial differential equations by finite difference scheme and calculating the integrals on the domain boundary. It is this integral who causes the computational complexity of the algorithm $\mathcal{O}(N^{4/3})$. To partially resolve the efficiency issue and to make the solver more flexible on handling the magnet with complicated geometry, we introduce an h-adaptive finite element method for the demagnetization field calculations. It can be observed from the numerical results that i). with the finite element discretization, the domain with curved defects can be resolved well, and ii). with the adaptive methods, the total amount of the mesh grids can be reduced significantly to reach the given accuracy, which effectively accelerates the simulations.

AMS subject classifications: 65M10

Key words: Computational micromagnetization, demagnetization field, single-layer potential, finite element methods, h-adaptive methods.

---

1 Introduction

Computational micromagnetics has been playing an important role in a variety of applications such as new data storage and manipulation [4, 12]. In the past decades, there

*Corresponding author.
Emails: leiyang@must.edu.mo (L. Yang), garyhu@umac.mo (G. H. Hu)

have been lots of pioneer works available towards the quality numerical simulations on the computational micromagneitcs, please refer to a recent topical review [11] and references therein.

In the development of the robust and efficient numerical methods for the computational micromagnetics, the quality calculation of the demagnetization field is one of the nontrivial challenges, not only because of the nonlocality of the evaluation, but also because of the irregularity of the computational domain from the practical simulations. For a given magnetization $\vec{M}$ on the domain Ω, the demagnetization field $\vec{H}_d$ is expressed by

$$\vec{H}_d(\vec{x}) = -\nabla\phi(\vec{x}), \quad \text{where} \quad \phi(\vec{x}) = \frac{1}{4\pi}\int_\Omega \vec{M}(\vec{x}')\cdot\nabla_{\vec{x}'}\left(\frac{1}{|\vec{x}-\vec{x}'|}\right)d\vec{x}'. \tag{1.1}$$

Here $\phi(\vec{x})$ is the demagnetization potential. For grid-based numerical methods, it is obvious that direct evaluation of the demagnetization potential $\phi(\vec{x})$ will cause $\mathcal{O}(N^2)$ computational complexity, where N denotes the total amount of the mesh grids. This is unaffordable for the practical simulations. To resolve the efficiency issue, the existing algorithms can be classified into two approaches, i.e., the fast summation approach and the partial differential equation (PDE) approach. The fast summation approach resorts to the fast Fourier transform method (FFT) for the structured mesh cases and the fast multipole method (FMM) for the unstructured mesh cases. The theoretical computational complexities for the FFT and FMM are $\mathcal{O}(N\log N)$ and $\mathcal{O}(\log(1/\varepsilon)N)$, respectively, where ε is a given tolerance representing the error. In [1,6], an interesting tensor grid (TG) method was introduced for the demagnetization calculation. TG method can be implemented on nonuniform mesh and the computational complexity is $\mathcal{O}(N^{4/3})$ and can be $\mathcal{O}(N^{2/3})$ when magnetization is given in canonical tensor format. These fast summation methods have been widely used in the numerical micromagnetics. However, the quality implementation of those methods heavily depend on the regularity of the computational domain as well as the magnetization property, which restrain their applications on the practical applications. Potentially, this issue can be effectively resolved by the PDE approach for the demagnetization field calculation.

Theoretically the demagnetization potential $\phi(\vec{x})$ in (1.1) is equivalent to the solution of the Poisson equation defined on $\mathbb{R}^3$,

$$\begin{cases} \Delta\phi(\vec{x}) = \nabla\cdot\vec{M}(\vec{x}), \\ \phi(\vec{x}) = 0 \quad \text{as} \quad |\vec{x}|\to\infty. \end{cases} \tag{1.2}$$

The idea of the PDE approach is to transform the above Poisson equation defined on $\mathbb{R}^3$ to the one defined on $\Omega\subset\mathbb{R}^3$, then the demagnetization potential $\phi(\vec{x})$ is obtained by numerically solving the equation. There have been several ways for such transformation. A straightforward approach is to use a sufficiently large domain containing the region Ω occupied by the magnet, then the regularity condition $\phi\propto 1/|\vec{x}|$ is imposed on the boundary of that larger domain. As a rule of thumb the distance between the boundary of the larger domain and the boundary of the magnet should be at least five times of the magnet