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Approximation of Smooth Stable Invariant Manifolds for Stochastic Partial Differential Equations

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Abstract. Invariant manifolds are complicate random sets useful for describing and understanding the qualitative behavior of nonlinear dynamical systems. The purpose of the present paper is try to approximate smooth stable invariant manifolds for a type of stochastic partial differential equations with multiplicative white noise near the fixed point. Two examples are given to illustrate our results.

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1 Introduction

Invariant manifolds are one of the most important invariant sets in nonlinear systems, they describe the dynamical behavior of the nonlinear systems completely. The theory of invariant manifolds for both finite-dimension and infinite-dimension autonomous deterministic systems, also for stochastic ordinary and partial differential equations are relatively mature. Stable, unstable, center, slow and inertial manifolds have been widely studied by many authors, see [1–12].

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Stochastic partial differential equations arise as macroscopic mathematical models of systems under random influences. There have been rapid progresses in this area, see [13–15], e.g., among others. More recently, stochastic partial differential equations have been investigated in the context of random dynamical systems see [7] and the references cited therein. Random invariant manifolds play an important role in the study of dynamic behavior of stochastic systems, they provide a geometric structure to understand or reduce stochastic dynamical systems. There are many works about the invariant manifolds for stochastic differential equations and stochastic partial differential equations. For example, Mohammed and Scheutzow [15] considered the stable and unstable manifolds for stochastic differential equations driven by semi-martingales. Boxler [2], Chen [16] considered the center manifolds for stochastic differential equations on finite-dimension space and infinite-dimension space. Duan and co-authors [14, 17] considered invariant manifolds for stochastic partial differential equations under the assumption of exponential dichotomy or pseudo exponential dichotomy, they used the Lyapunov-perron method to construct the existence of invariant manifolds.

In the present paper, we consider the approximation of invariant stable manifolds for a type of stochastic partial differential equations as following

$$\frac{\mathrm{du}}{\mathrm{dt}} = Au + F(u) + \sigma u \circ \dot{W}. \tag{1.1}$$

Compared with the unstable manifolds case in paper [18], our assumption on the nonlinearity is more generally. Along with the gap condition of linear operator A, we derive the approximation result of smooth stable invariant manifolds for this type of stochastic partial differential equations, similar work also consider by other authors. In paper [19], the authors considered approximation result of the local geometric shape of the unstable invariant manifold for a type of stochastic partial differential equation, with the quadratic nonlinearity, under this nonlinearity condition, the authors derive the second order convergence result. Compare with the work of [19], in [18], the authors considered the local geometric shape of the unstable invariant manifold case, but the nonlinearity takes as u^p, and the convergence order is one. However, in our paper, we take more generally condition on nonlinearity, and the convergence result is similar as [18].

This paper is organized as follows. In Section 2, we will review some basic concept of random dynamic systems, random evolutionary equation, assumption of the nonlinearity F and the linear operator A. In Section 3, we get the approximation of smooth stable invariant manifolds for the Eq. (1.1). In Section 4, we present two examples to illustrate our result.

2 Framework and preliminaries

Let H be a separable Hilbert space with norm $\|\cdot\|$ and scalar product $\langle\cdot,\cdot\rangle$. We assume