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On ∂-reducible 3-manifolds Which Admit Complete Surface Systems

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Abstract: In the present paper, we consider a class of compact orientable 3-manifolds with one boundary component, and suppose that the manifolds are ∂-reducible and admit complete surface systems. One of our main results says that for a compact orientable, irreducible and ∂-reducible 3-manifold M with one boundary component F of genus $n > 0$ which admits a complete surface system $\mathcal{S}'$, if $\mathcal{D}$ is a collection of pairwise disjoint compression disks for ∂M, then there exists a complete surface system $\mathcal{S}$ for M, which is equivalent to $\mathcal{S}'$, such that $\mathcal{D}$ is disjoint from $\mathcal{S}$. We also obtain some properties of such 3-manifolds which can be embedded in S^3.

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1 Introduction

Let $\mathcal{J} = \{J_1, \cdots, J_n\}$ be a collection of pairwise disjoint simple closed curves on a connected orientable closed surface S of genus n. If the surface obtained by cutting S open along $\mathcal{J}$ is a $2n$-punctured 2-sphere, we call $\mathcal{J}$ a complete curve system (or simply, CCS). Two CCSs on S are equivalent if one can be obtained from another via finite number of band moves (defined in Section 2) and isotopies.

Let M be a compact orientable 3-manifold with one boundary component F, and $\mathcal{S} = \{S_1, \cdots, S_n\}$ a collection of n pairwise disjoint connected orientable surfaces properly embedded in M. If $\partial\mathcal{S} = \{\partial S_1, \cdots, \partial S_n\}$ is a CCS on F, we call $\mathcal{S}$ a complete surface system (or simply, CSS) for M. Two CSSs $\mathcal{S}_1$ and $\mathcal{S}_2$ for M are equivalent if $\partial\mathcal{S}_1$ and $\partial\mathcal{S}_2$ are equivalent on F.

It is well known that any two complete disk systems in a handlebody are equivalent, that

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is, the equivalent classes of complete disk systems for a handlebody are unique. Clearly, a complete system of disks for a handlebody H_n of genus n is a CSS for H_n. By Corollary 1.4 in [1], the complete disk systems are the only CSSs in a handlebody H_n.

In this paper, we consider a class of compact orientable 3-manifolds with one boundary component, and suppose that the manifolds are ∂-reducible and admit CSSs. One of our main results says that for a compact orientable, irreducible and ∂-reducible 3-manifold M with one boundary component F of genus $n > 0$ which admits a CSS $\mathcal{S}'$, if $\mathcal{D}$ is a collection of pairwise disjoint compression disks for ∂M, then there exists a CSS $\mathcal{S}$ for M, which is equivalent to $\mathcal{S}'$, such that $\mathcal{D}$ is disjoint from $\mathcal{S}$. We also obtain some properties of such 3-manifolds which can be embedded in S^3.

The paper is organized as follows. Section 2 contains some necessary preliminaries. In Section 3, we prove one of the main results mentioned as above, and in Section 4, we obtain some properties of such 3-manifolds which are 3-submanifolds of S^3.

2 Preliminaries

The terminology and definitions used in the paper are all standard, see for example, refer to [2]–[3].

2.1 Complete Surface Systems for 3-manifolds

In this part, we introduce some definitions on complete surface systems for 3-manifolds.

Definition 2.1 *Let $S = S_n$ be a closed orientable surface of genus n. A collection of n pairwise disjoint simple closed curves $\mathcal{J}$ on S is called a complete curve system (or simply, CCS) for S if the surface obtained by cutting S open along $\mathcal{J}$ is a $2n$-punctured sphere.*

Definition 2.2 *Let $S = S_n$ be a closed orientable surface of genus n, $n \geq 2$.*

(1) *Let J_1, J_2 be two disjoint essential simple closed curves on S. Let γ be a simple arc on S with one endpoint lying in J_1, and another endpoint lying in J_2, and the interior of γ is disjoint from J_1 and J_2. Let $P = N(J_1 \bigcup \gamma \bigcup J_2)$ be a compact regular neighborhood of $J_1 \bigcup \gamma \bigcup J_2$ on S. Denote the component of ∂N, which is parallel to neither J_1 nor J_2 on P, by $J_1 \#_\gamma J_2$, and call it the band sum of J_1 and J_2 along γ.*

(2) *Let $\mathcal{J} = \{J_1, \cdots, J_n\}$ be a CCS on S. For $1 \leq i < j \leq n$, let γ be a simple arc on S with one endpoint lying in J_i, and another endpoint lying in J_j, and the interior of γ is disjoint from $\bigcup\limits_{1 \leq k \leq n} J_k$. Let J_{ij} be the band sum of J_i and J_j along γ. By isotopy, we may assume that J_{ij} is disjoint from the curves in $\mathcal{J}$. Set $\mathcal{J}' = (\mathcal{J} \setminus J_i) \bigcup \{J_{ij}\}$ or $(\mathcal{J} \setminus J_j) \bigcup \{J_{ij}\}$. It is easy to see that $\mathcal{J}'$ is still a CCS for S. We call $\mathcal{J}'$ an elementary band move of $\mathcal{J}$ along γ.*

(3) *Two CCSs $\mathcal{C}_1$ and $\mathcal{C}_2$ on S are called equivalent if one can be obtained from another by a finite number of elementary band moves and isotopies.*