Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2018-0054

Vol. **27**, No. 2, pp. 460-479
February 2020

Upwind Biased Local RBF Scheme with PDE Centres for the Steady Convection Diffusion Equations with Continuous and Discontinuous Boundary Conditions

K. Monysekar and Y. V. S. S. Sanyasiraju*

Department of Mathematics, Indian Institute of Technology Madras, Chennai, 600036, India.

Received 3 March 2018; Accepted (in revised version) 8 November 2018

Abstract. RBF based grid-free scheme with PDE centres is experimented in this work for solving Convection-Diffusion Equations (CDE), a simplified model of the Navier-Stokes equations. For convection dominated problems, very few integration schemes can give converged solutions for the entire range of diffusivity wherein sharp layers are expected in the solutions and accurate computation of these layers is a big challenge for most of the numerical schemes. Radial Basis Function (RBF) based Local Hermitian Interpolation (LHI) with PDE centres is one such integration scheme which has some built in upwind effect and hence may be a good solver for the convection dominated problems. In the present work, to get convergent solutions consistently for small diffusion parameters, an explicit upwinding is also introduced in to the RBF based scheme with PDE centres, which was initially used to solve some time dependent problems in [10]. RBF based numerical schemes are one type of grid free numerical schemes based on the radial distances and hence very easy to use in high dimensional problems. In this work, the RBF scheme, with different upwind biasing, is used to a variety of steady benchmark problems with continuous and discontinuous boundary data and validated against the corresponding exact solutions. Comparisons of the solutions of the convective dominant benchmark problems show that the upwind biasing either in source centres or PDE centres gives convergent solutions consistently and is stable without any oscillations especially for problems with discontinuities in the boundary conditions. It is observed that the accuracy of the solutions is better than the solutions of other standard integration schemes particularly when the computations are carried out with fewer centers.

AMS subject classifications: 76D99, 65N06, 65N35

Key words: Grid-free scheme, radial basis function, convection-diffusion, multi-quadric, PDE points, upwind.

*Corresponding author. *Email addresses:* `k.monys@gmail.com` (K. Monysekar), `sryedida@iitm.ac.in` (Y. V. S. S. Sanyasiraju)

1 Introduction

Grid free integration schemes are important when it becomes tedious and time consuming to generate meshes in the solution domain, particularly, when the geometry is complicated. In grid free schemes, the solution is approximated over a set of solution centres which are scattered in the problem domain with no specified connectivity between them. There are many grid free integration schemes with their own pros and cons. Radial Basis Function (RBF) based grid free schemes have been widely being used in recent years due to higher dimensional problems. Many investigators have carried out detailed analysis on grid free schemes based on Radial Basis Functions (RBFs) for solving Partial Differential Equations (PDEs) and successfully investigated the reconstruction of functions and their derivatives from multivariate scattered data.

RBF collocation was used in a global way by Kansa [1] for solving PDEs. In many occasions, the collocation matrix obtained by this method is non-symmetric and singular. Fasshauer [2] and Wu [3] used RBFs with Hermite-interpolation and the collocation matrix obtained is symmetric and non-singular. Although the global RBF methods are flexible and exhibit a higher order of convergence, they produced ill-conditioned systems especially when the size of the data set is very large. Many improvements have been proposed in the literature to avoid ill-conditioned systems. Local approximation, as in Chandhini and Sanyasiraju [4] and Sanyasiraju and Chandhini [5], is one such improvement. In these works, the operator approximations are realized using a very small local support in the neighbourhood of the solution centre. This scheme is similar to the finite difference type scheme proposed by [6], when applied over structured and uniform points.

In these grid-free local schemes, the discretization of the derivatives at any particular point is obtained locally using radial basis functions over a small set of neighbouring points without any physical connectivity between them. The advantage of such a local method is that the individual RBF systems never grow large and the method can be applied to large data sets without any numerical ill conditioning. This method also retains the flexibility of working on a set of global scattered data at the expense of spectral convergence. Stevens et al. [7] and [8] used PDE centres along with the source centres in the RBF based Local Hermite Interpolation (LHI) to improve the numerical solution. A PDE centre is a data centre at which the given governing equation is satisfied in a discrete sense.

In an earlier work [10], the authors have proposed an upwind biased scheme based on the approximation of RBFs with PDE centres to solve the unsteady Convection-Diffusion equations and validated their results with some benchmark problems with smooth boundary conditions. This upwind scheme includes the boundary operator within the basis functions and allows for the natural description of the boundary conditions. The inclusion of the PDE operator in the basis functions will help by exactly satisfying the governing PDEs at some specified locations. After writing the unknown variable in terms of its values at the neighbouring points, the governing equations are satisfied at each