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Existence of Solutions for Fractional Differential Equations with Conformable Fractional Differential Derivatives

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Abstract: A class of nonlinear fractional differential equations with conformable fractional differential derivatives is studied. Firstly, Green's function and its properties are given. Secondly, some new existence and multiplicity conditions of positive solutions are obtained by the use of Leggett-Williams fixed-point theorems on cone.
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1 Introduction

In this paper, we are concerned with the existence of three positive solutions to the following nonlinear conformable fractional differential equation boundary value problem (BVP)

$$D^{\alpha}u(t) + f(t, u(t)) = 0, \quad 0 < t < 1, \tag{1.1}$$

$$u(0) = u(1) = 0, \tag{1.2}$$

where $1 < \alpha \leq 2$ is a real number, D^{α} is the conformable fractional derivative, and $f\colon [0, 1]\times [0, +\infty) \to [0, +\infty)$ is a continuous function. One of the difficulties here is the corresponding Green's function $G(t, s)$ is singular at $s = 0$. Based on the Leggett-Williams fixed point theorem on cone, we obtain sufficient conditions for the existence of three positive solutions to the conformable fractional boundary value problem (1.1)-(1.2).

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Recently, the existence of positive solutions to the fractional differential equations has been of great interest, this is caused both by the intensive development of the theory of fractional calculus itself and by the varied applications in many fields of science and engineering, see, for instance, [1]–[3] and the references therein.

Bai and Lü[4] considered the positive solutions for boundary value problem of nonlinear fractional differential equation with Riemann-Liouville fractional derivative, i.e.,

$$D_{0+}^{\alpha}u(t) + a(t)f(t,\, u(t)) = 0, \qquad 0 < t < 1,\ 1 < \alpha \leq 2, \tag{1.3}$$

$$u(0) = u(1) = 0, \tag{1.4}$$

by defining Green's function and using Krasnoselskii fixed point theorem, the existence and multiplicity of positive solutions of (1.3)-(1.4) are acquired. Kang *et al.*[5] studied the existence of three positive solutions to the following boundary value problem of nonlinear fractional differential equation

$$D_{0+}^{\alpha}u(t) + \lambda h(t)f(u(t)) = 0, \qquad 0 < t < 1,\ 3 < \alpha \leq 4, \tag{1.5}$$

$$u(0) = u'(0) = u''(0) = u''(1) = 0, \tag{1.6}$$

where $\lambda > 0$ is a positive parameter and D_{0+}^{α} is the standard Riemann-Liouville fractional derivative, $h\colon (0,\, 1) \to (0,\, \infty)$ is continuous with $\int_0^1 h(t)\mathrm{d}t > 0$, and $f\colon [0,\, \infty) \to [0,\, \infty)$ is continuous. Based on the Leggett-Williams fixed point theorem, some results for the existence of three positive solutions to the fractional boundary value problem (1.5)-(1.6) are obtained.

Afterwards, these methods attract many authors, and apply in all kinds of fractional differential equation boundary value problems, such as Riemann-Liouville fractional derivative problems (see [6]), Caputo fractional derivative problem (see [7]), impulsive problems (see [8]), integral boundary value problems (see [9]), etc.

In recent years, a new conformable fractional derivative was defined (see [10]–[12]), Dong and Bai[13] established the existence of positive solutions for nonlinear eigenvalue problems and some boundary value problems with conformable fractional differential derivatives. This motivated our study interesting with problem (1.1)-(1.2) by this new derivatives.

Duo to the corresponding Green's function $G(t,\, s)$ is singular at $s = 0$, to the knowledge of the author, there are very few works on the existence and multiplicity of positive solutions of boundary value problems for above nonlinear fractional differential equation.

To ensure that readers can easily understand the results, the rest of the paper is planned as follows. In Section 2, we recall certain basic definitions and present some lemmas. In Section 3, by the use of approach method and Krasnoselkii's fixed points theorem on cone we obtain that the existence conditions of BVP (1.1)-(1.2) has at least one positive solution or three positive solutions. In Section 4, we give an example as an application of our results.