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PERMANENCE OF A DISCRETE LOGISTIC EQUATION WITH PURE TIME DELAYS*†

Yangyang Lü[1], Lijuan Chen[1]‡, Liujuan Chen[2]

(1. *College of Math. and Computer Science, Fuzhou University, Fuzhou 350116, Fujian, PR China*)

2. *Department of Mathematics Research and Training, Fujian Institute of Education, Fuzhou 350025, Fujian, PR China*)

Abstract

In this paper we propose a discrete logistic system with pure delays. By giving the detail analysis of the right-hand side functional of the system, we consider its permanence property which is one of the most important topic in the study of population dynamics. The results obtained in this paper are good extensions of the existing results to the discrete case. Also we give an example to show the feasibility of our main results.

Keywords permanence; discrete; delay; logistic system

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1 Introduction

Recently, lots of scholars have investigated the logistic equation. For example, in [1], a sufficient condition was obtained for the existence and the attractivity of a unique almost periodic solution by constructing suitable Lyapunov functional and almost periodic functional hull theory. The authors [2] discussed the permanence and the global attractivity of the model, based on the boundedness of solutions of the corresponding autonomous logistic model. As we know, the traditional single species logistic equation takes the form:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = N(t)\big(a - bN(t)\big). \tag{1.1}$$

Many scholars argued that a more suitable model should include some past state of the system, thus, a more suitable single species model should take the form:

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‡Corresponding author. E-mail: chenlijuan@fzu.edu.cn

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = N(t)\Big(a - bN(t) - cN(t-\tau) - d\int_0^\infty K(s)N(t-s)\mathrm{d}s\Big). \tag{1.2}$$

Here, a is a positive constant; b, c, d are all nonnegative constants and at least one of them is a positive constant. We divide system (1.2) into the following two cases:

(1) $b \equiv 0$, in this case, we call the system pure time delay system;

(2) $b \neq 0$, in this case, we call the system non-pure time delay system.

For the second case, by constructing a suitable Lyapunov function, one could easily show that condition $b > c + d$ is enough to ensure the existence of unique globally attractive positive equilibrium. However, the dynamic behaviors of the first case is very complicated. Indeed, for the infinite delay case, that is, $b = c = 0$ in system (1.2), under the assumption that $a(t)$ and $d(t)$ are all continuous ω-period functions, Gopalsamy [3] studied the following logistic system:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = N(t)\Big[a(t) - d(t)\int_0^\infty K(s)N(t-s)\mathrm{d}s\Big], \tag{1.3}$$

and investigated the existence and uniqueness of the periodic solution of system (1.3). The main results of [3] is then generalized by Seifert [4] to the almost periodic case, but require a somewhat stronger assumption on the decay kernel $K(t)$, that is,

$$\int_0^\infty K(s)\mathrm{d}s = 1, \quad \int_0^\infty \mathrm{e}^{rs}K(s)\mathrm{d}s < \infty, \tag{1.4}$$

where r is some positive constant.

Feng [5] extended the main results of Seifert [4] to the following almost periodic system with delays:

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = N(t)\Big[a(t) - c(t)N(t-\tau) - d(t)\int_0^\infty K(s)N(t-s)\mathrm{d}s\Big]. \tag{1.5}$$

Feng also weakened condition (1.7) to the following condition:

$$\int_0^\infty K(s)\mathrm{d}s = 1, \quad \int_0^\infty s^2K(s)\mathrm{d}s < \infty. \tag{1.6}$$

Meng, Chen and Wang [6] further weakened condition (1.6) to the following condition:

$$\int_0^\infty K(s)\mathrm{d}s = 1, \quad \int_0^\infty sK(s)\mathrm{d}s < \infty. \tag{1.7}$$

By using almost periodic functional Hull theory and some computational techniques, Meng et al. [6] showed that condition (1.7) together with some other conditions is enough to ensure the boundedness and global asymptotically attractivity of system (1.5). For the case $b = d = 0$ in system (1.2), that is, for the system