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THE STABILITY OF A PREDATOR-PREY MODEL WITH FEAR EFFECT IN PREY AND SQUARE ROOT FUNCTIONAL RESPONSE*

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Abstract

In this paper, we consider a predator-prey model with fear effect and square root functional response. We give the singularity of the origin and discuss the stability and Hopf bifurcation of the trivial equilibrium and the positive equilibrium. We show that the fear effect has no effect on prey density, but will lead to the decrease of predator populations.

Keywords predator-prey; fear effect; stability; Hopf bifurcation
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1 Introduction

Based on some experimental studies, [1,2] showed that the cost of fear can change the anti-predator defences, thus can greatly reduce the reproduction of prey. Hence, Wang et al. [3] proposed a predator-prey model incorporating the cost of fear into prey reproduction as follows

$$\begin{cases} x'(t) = \dfrac{r_0 x}{1+ky} - dx - ax^2 - \dfrac{pxy}{1+qy}, \\ y'(t) = \dfrac{cpxy}{1+qy} - my, \end{cases} \tag{1.1}$$

where r_0 is the birth rate of prey; d is the natural death rate of prey; k is the level of fear. They studied the stability and Hopf bifurcation of the system, and showed that the existence of Hopf bifurcation is different from that of model (1.1) without fear effect. Duan et al. [4] discussed a diffusive predator-prey model by incorporating the fear effect into prey, and found that time delay makes the dynamics behaviour of the predator-prey system more complicated. Zhang et al. [5] investigated the stability of a predator-prey system with prey refuge and fear effect. Xiao and Li [6] showed

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that the fear effect has no influence on the stability of system. Pal et al. [7] studied the stability of a predator-prey model with fear effect in prey and hunting cooperation, and showed two different types of bi-stabilities behaviour.

Different from the model in [3], Sasmal [8] proposed the following predator-prey model with fear effect and Allee effect in prey

$$\begin{cases} x'(t) = rx\Big(1-\dfrac{x}{k}\Big)(x-\theta)\dfrac{1}{1+fy} - axy, \\ y'(t) = a\alpha xy - my, \end{cases} \tag{1.2}$$

where $0<\theta<k$ is Allee effect and f is the level of fear. Due to the fear effect, the intrinsic growth rate of prey is modified by $\frac{r}{1+fy}$, which is a monotonic decreasing function of both f and y. They showed the fear can greatly affect the stability of system.

Braza [9] considered a predator-prey model with a modified Lotka-Volterra interaction term, which is proportional to the square root of the prey population. Since the square root term, the dynamic behavior of the origin is more subtle. N. Fakhry and R. Naji [10] investigated the following predator-prey system with fear effect and square root function response

$$\begin{cases} x'(t) = rx(1-x)\dfrac{1}{1+ky} - \sqrt{x}y, \\ y'(t) = -\alpha y + \beta\sqrt{x}y, \end{cases} \tag{1.3}$$

where r is the growth rate of the prey; α is the death rate of the predator in the absence of prey; β is the conversion rate of prey to predator; k is the level of fear. It is easy to deduce that there exist a trivial equilibrium $E_0(0,0)$ and a boundary equilibrium $E_1(1,0)$. If $\beta>\alpha$, there exists a positive equilibrium $E^*\,(x^*,y^*)$, where

$$x^* = \frac{\alpha^2}{\beta^2}, \quad y^* = \frac{-1+\sqrt{1+4k\dfrac{r\alpha}{\beta^3}\left(\beta^2-\alpha^2\right)}}{2k}.$$

They obtained the following conclusions.

Proposition A (1) *E_0 is a saddle point.*

(2) *If $\beta<\alpha$, then the trivial equilibrium $E_1(1,0)$ is locally asymptotically stable.*

(3) *If $\frac{y^*}{2\sqrt{x^*}} > \frac{r-2rx^*}{1+ky^*}$, then the positive equilibrium E^* is locally asymptotically stable.*

Theorem A (1) *Assume that the trivial equilibrium E_1 is locally asymptotically stable. If $\sqrt{x} > \frac{\beta}{\alpha}$, then E_1 is globally asymptotically stable.*

(2) *Assume that the trivial equilibrium E^* is locally asymptotically stable. If $x<\frac{1}{3}$ or $x>\frac{1}{3}$, then E^* is globally asymptotically stable.*