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IMAGE RESTORATION UNDER CAUCHY NOISE WITH SPARSE REPRESENTATION PRIOR AND TOTAL GENERALIZED VARIATION*

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Abstract

This article introduces a novel variational model for restoring images degraded by Cauchy noise and/or blurring. The model integrates a nonconvex data-fidelity term with two regularization terms, a sparse representation prior over dictionary learning and total generalized variation (TGV) regularization. The sparse representation prior exploiting patch information enables the preservation of fine features and textural patterns, while adequately denoising in homogeneous regions and contributing natural visual quality. TGV regularization further assists in effectively denoising in smooth regions while retaining edges. By adopting the penalty method and an alternating minimization approach, we present an efficient iterative algorithm to solve the proposed model. Numerical results establish the superiority of the proposed model over other existing models in regard to visual quality and certain image quality assessments.

Mathematics subject classification: 68U10, 65K10.
Key words: Image restoration, Cauchy noise, Sparse representation prior, Dictionary learning, Total generalized variation.

1. Introduction

Image restoration is a fundamental problem in image processing. It refers to estimating the original clean image from an observed image. This work focuses on image denoising and/or deblurring problem in the presence of Cauchy noise. The Cauchy noise is a type of impulsive noise that emerges in atmospheric and underwater acoustic noise, radar and sonar applications, wireless communication systems, biomedical images, and synthetic aperture radar images; see for instance [1–3].

Let $u \in \mathbb{R}^{m\times n}$ be a true 2-dimensional (2D) discrete grayscale image. Assume that the observed image $f \in \mathbb{R}^{m\times n}$ is given by

$$f = Au + \eta, \tag{1.1}$$

where $A : \mathbb{R}^{m\times n} \to \mathbb{R}^{m\times n}$ represents either the identity operator or a blurring operator, defined as $Au = \kappa * u$ with κ as a blurring kernel and $*$ denoting the convolution, and η represents

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some Cauchy noise. That is, η is a random variable following a Cauchy distribution with the probability density function (PDF) [4, 5] as follows

$$P(\eta) = \frac{\gamma}{\pi\Big(\gamma^2 + (\eta - \delta)^2\Big)}, \tag{1.2}$$

where $\delta \in \mathbb{R}$ is the parameter specifying the location of the peak, and $\gamma > 0$ is the scale parameter that determines the level of noise. Here, we intend to retrieve a clean image u from an observed image f, which is an ill-posed inverse problem.

In recent years, several approaches to eliminating Cauchy noise have been suggested. Chang et al. [6] proposed a recursive restoration algorithm based on a Markov random field (MRF) model driven by Cauchy noise, and the proposed method was shown to provide better edge preservation than similar algorithms using Gaussian or Laplacian noise. In [7], the authors derived a new statistical model in the complex wavelet domain to remove Cauchy noise, from a bivariate *maximum a posteriori* (MAP) estimator. In addition, Loza et al. [8] proposed a statistical image fusion method based on a non-Gaussian distribution in the wavelet domain, which showed a noticeable improvement in terms of fusion quality and noise reduction. In [9], Wan et al. introduced a novel image segmentation method for noisy color images corrupted by Cauchy noise. The authors developed a noise reduction approach in the wavelet domain, relying on the bivariate Cauchy density, that was utilized for segmentation.

In addition to these MRF or wavelet-based denoising methods, a variational model was proposed in [10] to restore images degraded by Cauchy noise. This model involves a total variation (TV) regularization [11] and a nonconvex data-fidelity term derived from the PDF in (1.2). Specifically, assuming that η follows a *zero-centered* Cauchy law ($\delta = 0$), the authors proposed a TV model for restoring images corrupted by Cauchy noise as follows:

$$\min_u \frac{\lambda}{2}\langle \log(\gamma^2 + (Au - f)^2), \mathbf{1}\rangle + \mathrm{TV}(u), \tag{1.3}$$

where $\lambda > 0$ is a tuning parameter, $\langle\, , \rangle$ represents the inner product, and TV denotes the discrete version of the isotropic TV norm: denoting u_s by the pixel value of an image u at location $s = (i, j)$ $(i = 1, \ldots, m,\ j = 1, \ldots, n)$, $\mathrm{TV}(u) = \|\nabla u\|_1 = \sum_s \sqrt{|(\nabla u)^1_s|^2 + |(\nabla u)^2_s|^2}$, where $\nabla u = [\partial_x u, \partial_y u]^T$ is a discrete gradient operator whose components $\partial_x u$ and $\partial_y u$ are the finite difference operators that estimate the partial derivatives of the image u along the x-axis and y-axis, respectively. The TV enabled the recovery of images with well-preserved structures and important edges. However, due to the deficiency of the nonconvexity, the same authors introduced a convex model by adding a quadratic penalty term, $\|u - \tilde{u}\|_2^2$, into the nonconvex model. This quadratic term contains a pre-denoised image, $\tilde{u}$, obtained by applying the median filter to the data f. However, the median filtering does not always bring adequate denoising results. Recently, Mei et al. [12] returned to the original nonconvex problem (1.3), and showed the effectiveness of the model combined with the alternating direction method of multipliers (ADMM) [13].

Despite its several benefits, TV regularization has a tendency to produce staircase artifacts in smoothly varying regions, as it favors solutions that are piecewise constant, and thus it smoothes textures and fine details in images. Thus, as one way of ameliorating staircasing effects, higher-order regularization based models were suggested in [14–18]. As an early work, a inf-convolution TV (ICTV) was proposed in [14], which takes the infimal convolution of TV and second-order TV. Moreover, Li et al. [17] proposed a denoising model, involving a convex