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Mortar DG Method with Staggered Hybridization for Rayleigh Waves Simulation

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Abstract. The simulation of Rayleigh waves is important in a variety of geophysical applications. The computational challenge is the fact that very fine mesh is necessary as the waves are concentrated at the free surface and decay exponentially away from the free surface. To overcome this challenge and to develop a robust high order scheme for the simulation of Rayleigh waves, we develop a mortar discontinuous Galerkin method with staggered hybridization. The use of the mortar technique allows one to use fine mesh in only a local region near the free surface, and use coarse mesh in most of the domain. This approach reduces the computational cost significantly. The staggered hybridization allows the preservation of the strong symmetry of the stress tensor without complicated construction of basis functions. In particular, the basis functions are piecewise polynomial without any continuity requirement, and the coupling of the basis functions is performed by using carefully chosen hybridized variables. The resulting scheme is explicit in time, and only local saddle point system are solved for each time step. We will present several benchmark problems to demonstrate the performance of the proposed method.

AMS subject classifications: 65M32, 65M60

Key words: Discontinuous Galerkin method, elastic wave equations, Rayleigh wave, mortar formulation, hybridization.

1 Introduction

Accurate elastodynamic simulations are of critical importance in a variety of geophysical applications. The staggered grid finite difference methods [21,23,29] is a class of efficient numerical schemes for accurate elastic wave computations, and they are widely used in

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a number of applications. For computational domains with irregular geometries, such as non-flat topography, the accuracy of these methods diminishes. For many realistic applications, one needs accurate and efficient computational techniques that can be applied to domains with complex geometries or non-flat interfaces. The discontinuous Galerkin (DG) methods can be used to tackle these computational challenges. The DG method approximates the solution by using piecewise polynomial functions defined on unstructured meshes, which can triangulate complex domain geometries. For example, various DG methods are proposed in [1, 4, 13–16, 18, 19, 22, 24, 25, 27, 28, 30].

The use of staggered mesh for computational wave propagation has shown its prominence in applications. Motivated by the staggered grid finite difference schemes, the staggered discontinuous Galerkin (SDG) method is developed with the goal of achieving high order accuracy on domains with irregular geometries and keeping the advantages of using staggered meshes. The SDG methods are successfully applied to both the acoustic wave equations [7, 8, 20] and the elastic wave equations [6, 9] as well as other applications [3, 17, 31]. The use of staggered mesh in discontinuous Galerkin method offers several additional advantages such as energy conservation, optimal rate of convergence and low dispersion error [2, 9]. Another key feature of the proposed method in this paper is the use of staggered hybridization [6]. This staggered hybridization techniques allows one to define the polynomial basis functions locally on each cell in the mesh without enforcing any continuity condition. The coupling of the basis functions is defined by using suitable staggered hybridized variables. One advantage of this technique is that the symmetry of the stress tensor can be enforced strongly on irregular meshes. We remark that the idea of hybridization has been used successfully in discontinuous Galerkin methods [5, 10–12, 25, 26]. The technique of staggered hybridization shares many of the advantages of hybridization, such as superconvergence, and gives additional advantages for elastic wave simulations as mentioned above. Furthermore, the proposed method gives explicit time-stepping scheme. In particular, one needs only to solve local saddle point system for each time step. So, the time-stepping is very efficient.

The focus of this paper is efficient simulations of Rayleigh waves. Accurate and efficient computations of Rayleigh waves has important applications in geophysics. From the computational point of view, the simulation of Rayleigh waves is difficult in the sense that a very fine mesh is necessary as the wave is concentrated only on the free surface and decays exponentially in the direction away from the free surface. We also remark that a fine mesh in the whole computational domain is needed even though the wave is only concentrated near the surface. To overcome this computational challenge, we propose the use of mortar technique together with our discontinuous Galerkin method using staggered hybridization. The main idea is to use a fine computational mesh near the surface of the domain, and use a coarse mesh in the rest of the domain. The region of fine mesh is a thin layer near the surface. Thus, the overall degrees of freedom in the whole domain is much reduced. In order to couple the unknown at the interface of the fine and the coarse meshes, we apply the mortar technique. More precisely, we define an additional mortar variable and an additional jump condition to enforce the continuity of