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Unconditional Stability and Error Estimates of the Modified Characteristics FEM for the Time-Dependent Viscoelastic Oldroyd Flows

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Abstract. In this paper, our purpose is to study the unconditional stability and convergence of characteristics finite element method (FEM) for the time-dependent viscoelastic Oldroyd fluids motion equations. We deduce optimal error estimates in L^2 and H^1 norm. The analysis is based on an iterated time-discrete system, with which the error function is split into a temporal error and a spatial error. Finally, numerical results confirm the theoretical predictions.

AMS subject classifications: 76M10, 65N12, 65N30, 35K61

Key words: Unconditional stability, optimal error estimates, modified characteristics finite element method, time-dependent viscoelastic Oldroyd flows.

1 Introduction

In this paper, we consider the time-dependent viscoelastic Oldroyd flows, which are governed by the initial-boundary value problem [10,19]

$$\begin{cases} u_t-\nu\Delta u-\int_0^t \beta(t-s)\Delta u(x,s)ds+(u\cdot\nabla)u+\nabla p=f, & x\in\Omega\times[0,T], \\ \nabla\cdot u=0, & x\in\Omega\times[0,T], \\ u(x,0)=u_0(x), & x\in\Omega, \\ u(x,t)=0, & x\in\partial\Omega\times[0,T], \end{cases} \tag{1.1}$$

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where $\beta(t)=\rho\exp(-\delta t)$, $1/\delta$ is the relaxation time, $\rho\geq 0$ is the viscoelastic coefficient number. Ω is a bounded domain in $\mathbb{R}^2$ assumed to have a Lipschitz continuous boundary $\partial\Omega$. Here, $u=(u_1(x,t),u_2(x,t))$ represents the velocity vector, $p(x,t)$ is the pressure, $f(x,t)$ is the body force. When $\rho=0$, the system reduces to the Navier-stokes flows.

The method of characteristic type springs from a finite difference approximation about particles in cells, see [14], which is applied to linear systems of PDEs and to nonlinear PDE problems. What's more, it's also important to solve the convection-dominated case. In the method, the hyperbolic part (the temporal and advection term) is approximated by a characteristic tracking scheme, such as

$$(u_t+b\cdot\nabla u)|_{t=t^{n+1}}\approx\frac{u^{n+1}(x)-u^n(x-b^n\tau)}{\tau}, \tag{1.2}$$

which can obtain more precise approximation in practice. Because of the effort of authors, the modified characteristics method, which combines finite difference with finite element approximations, was developed by Douglas and Russell [11]. Recently, some authors have studied the high order time discrete schemes based on the methods of characteristics [5, 6, 21]. In addition, there are other kinds of methods of characteristics in [3,7,8,12,17,20,23].

The theory of viscoelastic fluids developed in the second half of the 20th century with the industrial production of molten and dilute polymers and the growth of engineering that generated many new products. Numerical modeling of viscoelastic flows is of great importance for complex engineering applications involving foodstuff, blood and paints. The existence, uniqueness, and continuous dependence of the solution were studied in [4, 10]. The problem (1.1) has been extensively studied. As a decoupling method, the penalty method was presented by Oskolkov [18], and Wang et al. [24, 27]. In [19], Pani et al. considered a linearized backward Euler semi-discrete scheme. Wang et al. deduced the long time numerical stability of the viscoelastic flow in [25]. In [26], a fully discrete scheme and the optimal uniform-in-time error estimates under certain uniqueness assumption were obtained by Wang et al.

The outline of the paper is as follows. In Section 2, a functional setting of the problem is given together with main theoretical results. To acquire the unconditional stability and the full rate of convergence for velocity in Section 3, we introduce the characteristics time-discrete system and the dual technique. In order to test the method, Section 4 is concentrated on the numerical results.

2 Functional settings and the modified characteristics FEM

In the section, we briefly introduce a few notations. For $1\leq p\leq\infty$ and let m be a nonneg-