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A Highly Efficient Reduced-Order Extrapolating Model for the 2D Viscoelastic Wave Equation

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Abstract. We mainly research the reduced-order of the classical natural boundary element (CNBE) method for the two-dimensional (2D) viscoelastic wave equation by means of proper orthogonal decomposition (POD) technique. For this purpose, we firstly establish the CNBE model and analyze the existence, stability, and errors for the CNBE solutions. We then build a highly efficient reduced-order extrapolating natural boundary element (HEROENBE) mode including few degrees of freedom but possessing sufficiently high accuracy for the 2D viscoelastic wave equation by the POD method and analyze the existence, stability, and errors of the HEROENBE solutions by the CNBE method. We finally employ some numerical experiments to verify that the numerical results are accorded with the theoretical ones so that the validity for the HEROENBE model is further verified.

AMS subject classifications: 65N30, 65M30, 76M10
Key words: Highly efficient reduced-order extrapolating model, natural boundary element, proper orthogonal decomposition, viscoelastic wave equation, numerical experiments.

1 Introduction

Let $\Omega\subset R^2$ be a bounded and connected domain with smooth boundary $\Gamma=:\partial\Omega$ and $\Omega^c=:R^2\backslash\overline{\Omega}$. We take into account the following 2D viscoelastic wave equation in the unbounded outside domain Ω^c.

Problem 1.1. Find u such that

$$\begin{cases} u_{tt}-\gamma\Delta u_t-\varepsilon\Delta u=f(z,t), & (z,t)\in\Omega^c\times(0,T),\\ \dfrac{\partial u}{\partial \boldsymbol{n}}=g(z,t), & (z,t)\in\Gamma\times(0,T),\\ u(z,0)=v_0(z),\quad u_t(z,0)=v_1(z), & z\in\Omega^c, \end{cases} \tag{1.1}$$

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where $u_{tt}=\partial^2 u/\partial t^2$, $u_t=\partial u/\partial t$, $\Delta=\partial^2/\partial x^2+\partial^2/\partial y^2$, γ and ε are two given positive coefficients, T is the total time, $\partial/\partial \boldsymbol{n}$ is the exterior normal derivative, $\boldsymbol{n}$ is the unit exterior normal vector from boundary Γ of domain Ω^c toward Ω, $f(z,t)$ and $g(z,t)$ are, respectively, the given appropriate smooth source function and boundary function, $v_0(z)$ and $v_1(z)$ are the given appropriate smooth initial functions, $z=(x,y)$, and $|z|=\sqrt{x^2+y^2}$. Furthermore, we assume that the solution $u(z,t)$ is bounded at infinity just as that in [1,2].

Problem 1.1 is an important partial differential equation (PDE) with actual physical background. It can be used to describe the wave propagation phenomena of vibration in a viscoelastic medium (see [3]). However, it usually includes complex source function, initial function, and boundary value function so that we can not find its genuine solution and we have to depend on the numerical solutions.

The viscoelastic wave equation have been solved with various numerical methods, such as finite element (FE) method (see [4]), finite difference (FD) scheme (see [5]), finite volume element (FVE) method (see [6]), and collocation spectral method (see [7,8]). But these methods can only solve the bounded inner region problem. Whereas, Problem 1.1 with the unbounded outside domain Ω^c need usually to be solved by the CNBE method, which is established by Feng and Yu in the late 1970 (see [9–12]) and whose main thought consists in converting the PDE boundary value problem in the outside region into the discretized numerical model of integral equation on the boundary. The CNBE method has been used to solve various PDEs, including the hyperbolic equation, Sobolev equation, elliptic equation, and parabolic equation (see [1,2,9–13]).

When the outer region is very large, the partition points on the boundary would greatly increase, so that the unknowns and numerical integrals in the CNBE method would also greatly increase (see [13]). Thus, the round-off errors in the real-world CNBE calculations would accumulated quickly so as to happen floating-point overflow after some calculating steps and to be unable to get anticipated results. Therefore, under making sure the CNBE solutions holding sufficiently high accuracy, how to lessen the unknowns (degrees of freedom) in the CNBE method so as to retard the accumulation of round-off errors in the calculation and reduce the CPU run time is an urgently settled issue, which is the main objective in this paper.

The POD method is regarded as an effective and reliable reduced-order technique for the numeral model (see [14–17]). It has successfully been used to the reduced-order for the Galerkin method (see [18,19]), the FE method (see [20–22]), the FD scheme (see [23–25]), the FVE method (see [6,26]), and the spectral method (see [27–30]).

Nevertheless, as far as we know, there has not been any report that the HEROENBE model for the 2D viscoelastic wave equation in the unbounded domain is established by the POD technique. Therefore, we here establish the HEROENBE model only including a few unknowns by the POD technique. Particularly, we only employ the CNBE solutions at the initial a few time nodes as the snapshots to constitute the POD bases and establish the HEROENBE model for finding the HEROENBE solutions at all time nodes, which is equivalent to utilizing the available data (on the quite short time interval $[0,T_0]$, $T_0\ll T$) to