CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.2020-0031

Vol. **2**, No. 1, pp. 56-80
March 2021

Social Distancing as a Population Game in Networked Social Environments

Zhijun Wu*

Department of Mathematics, Iowa State University, Ames, Iowa 50011, USA.

Received 4 July 2020; Accepted 10 November 2020

Abstract. While social living is considered to be an indispensable part of human life in today's ever-connected world, social distancing has recently received much public attention on its importance since the outbreak of the coronavirus pandemic. In fact, social distancing has long been practiced in nature among solitary species, and been taken by human as an effective way of stopping or slowing down the spread of infectious diseases. Here we consider a social distancing problem for how a population, when in a world with a network of social sites, decides to visit or stay at some sites while avoiding or closing down some others so that the social contacts across the network can be minimized. We model this problem as a population game, where every individual tries to find some network sites to visit or stay so that he/she can minimize all his/her social contacts. In the end, an optimal strategy can be found for everyone, when the game reaches an equilibrium. We show that a large class of equilibrium strategies can be obtained by selecting a set of social sites that forms a so-called maximal r-regular subnetwork. The latter includes many well studied network types, which are easy to identify or construct, and can be completely disconnected (with $r=0$) for the most strict isolation, or allow certain degrees of connectivities (with $r>0$) for more flexible distancing. We derive the equilibrium conditions of these strategies, and analyze their rigidity and flexibility on different types of r-regular subnetworks. We also extend our model to weighted networks, when different contact values are assigned to different network sites.

AMS subject classifications: 92D30, 92D25, 91A22, 91A43, 90C35, 90C27

Key words: Social distancing, epidemic and pandemic prevention, population games, social networks, regular networks, optimal distancing strategies, rigidity, flexibility, and fragility of distancing strategies.

1 Introduction

Humans participate in all sorts of social activities, especially in modern societies. We go to school, go to work, attend meetings, go shopping, go to restaurants, watch shows,

*Corresponding author. *Email address:* zhijun@iastate.edu (Z. Wu)

movies, or sports, etc. The world is formed by a huge number of social sites that host these activities. These sites are also closely connected for people to contact, meet, and socialize across. The world is a well-connected network. People's daily life is simply a busy agenda of events at different event sites in this network, with different visiting frequencies for different sites. As such, the world can be modeled as a social network, with the nodes representing the social sites and the links the connections among the social sites. The social life of an individual can be described by the frequencies of the individual to visit these social sites, and the social behavior of the population can be described by these frequencies averaged over the whole population.

Mathematically, we can represent a social network with a graph $G=(V,E)$, where $V=\{1,\cdots,n\}$ is a set of nodes corresponding to the social sites of the network, and $E=\{(i,j):$ i and j connected$\}$ a set of links between the nodes representing the social connections among the social sites. Let $x\in R^n$, $x\geq 0$, $\Sigma_i x_i=1$, be a vector describing the social behavior of an individual on G, with x_i being the frequency of the individual to visit or stay at node i of G. Then, we can define a vector $y\in R^n$, $y\geq 0$, $\Sigma_i y_i=1$, for the social behavior of the whole population, with y_i being the average frequency of the population to visit or stay at node i of G. Let A be the adjacency matrix of G, $A_{i,j}=1$ if $(i,j)\in E$, $A_{i,j}=0$ if $(i,j)\notin E$, and $A_{i,i}=\alpha\in[0,1]$ for all $i=1,\cdots,n$. Then, in a y-population, the amount of social contacts an x-individual can make at node i must be $x_iA_{i,i}y_i$, the contacts with the population on node i, plus $x_iA_{i,j}y_j$, the contacts with the population on other nodes j, all together equal to $\Sigma_j x_iA_{i,j}y_j$. Across the network, the social contacts this individual can make must be $\Sigma_i\Sigma_j x_iA_{i,j}y_j=x^TAy$.

Consider x as the social strategy of an individual, and y the social strategy of the population. Consider $\pi(x,y)=x^TAy$ as a payoff function. We can then define a population game where each individual of the population tries to maximize his/her social payoff. The latter can be achieved when an optimal strategy x^* is found for every individual. The strategy for the population then becomes x^* as well, and a Nash equilibrium is reached. A Nash equilibrium of this game is thus a strategy x^* such that

$$\pi(x^*,x^*)\geq\pi(x,x^*),\quad \forall x\in S,$$

where $S=\{x\in R^n: \Sigma_i x_i=1,\ x_i\geq 0,\ i=1,\cdots,n\}$ is the set of all possible social strategies. This game is called a social networking game on network G, and has been discussed recently in Wang et al. in 2019 [1]. For social networking, interactions across different social sites are encouraged and the contacts among individuals in the same sites are discounted, and therefore, $A_{i,i}$ are set to zero for all $i=1,\cdots,n$.

Different from social networking, social distancing is to reduce social contacts instead. Therefore, for a given social network, social distancing can be considered as a problem to find a set of social sites in the network where interactions within and between these sites can be minimized. It can therefore be modeled as a population game where each individual tries to minimize his/her social contacts $\pi(x,y)=x^TAy$. A Nash equilibrium of this game is then a strategy x^* such that

$$\pi(x^*,x^*)\leq\pi(x,x^*),\quad \forall x\in S.$$