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Super-Resolution Inversion of Non-Stationary Seismic Traces

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Abstract. In reflection seismology, the inversion of subsurface reflectivity from the observed seismic traces (super-resolution inversion) plays a crucial role in target detection. Since the seismic wavelet in reflection seismic data varies with the travel time, the reflection seismic trace is non-stationary. In this case, a relative amplitude-preserving super-resolution inversion has been a challenging problem. In this paper, we propose a super-resolution inversion method for the non-stationary reflection seismic traces. We assume that the amplitude spectrum of seismic wavelet is a smooth and unimodal function, and the reflection coefficient is an arbitrary random sequence with sparsity. The proposed method can obtain not only the relative amplitude-preserving reflectivity but also the seismic wavelet. In addition, as a by-product, a special Q field can be obtained.

The proposed method consists of two steps. The first step devotes to making an approximate stabilization of non-stationary seismic traces. The key points include: firstly, dividing non-stationary seismic traces into several stationary segments, then extracting wavelet amplitude spectrum from each segment and calculating Q value by the wavelet amplitude spectrum between adjacent segments; secondly, using the estimated Q field to compensate for the attenuation of seismic signals in sparse domain to obtain approximate stationary seismic traces. The second step is the super-resolution inversion of stationary seismic traces. The key points include: firstly, constructing the objective function, where the approximation error is measured in L_2 space, and adding some constraints into reflectivity and seismic wavelet to solve ill-conditioned problems; secondly, applying a Hadamard product parametrization (HPP) to transform the non-convex problem based on the L_p $(0<p<1)$ constraint into a series of convex optimization problems in L_2 space, where the convex optimization problems are solved by the singular value decomposition (SVD) method and the regularization parameters are determined by the L-curve method in the case of single-variable inversion. In this paper, the effectiveness of the proposed method is demonstrated by both synthetic data and field data.

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1 Introduction

In reflection seismology, recovering the subsurface reflectivity from the observed seismic data lies in the core of target exploration. To retrieve the reflectivity, super-resolution (someone called high-resolution) inversion methods are one of the mainstream approaches. Following the research of [1], the super-resolution inversion can be treated as a process to retrieve the fine-scale structures of an object from coarse-scale information only. It may be more reasonable to use the term "seismic super-resolution" instead of the term "seismic high-resolution". Broadly speaking, the seismic deconvolution [2, 3], spectral whitening [4] and so on can be regarded as some kinds of super-resolution processing. In a narrow sense, the super-resolution inversion usually refers to the reflectivity inversion [5–7].

It should be pointed out that the convolution model is the theoretical basis of super-resolution inversion. Before super-resolution inversion, therefore, it is necessary to compensate for the seismic data because the field seismic data is non-stationary with amplitude absorption and phase dispersion caused by intrinsic an-elasticity of subsurface media [8–10]. To accomplish this, the Q values are required. Various methods have been proposed to estimate the Q values [11–15], such as the methods in the Fourier-frequency domain and the time-frequency domain. In the Fourier-frequency domain, the approaches usually include the logarithm spectral ratio, centroid frequency shift, and peak frequency shift methods. All amplitude spectra need to be calculated within a time window. For the reflection seismic data, it is full of challenges to properly select the window function and window length. To adaptively select a proper window, [16] proposed a method to divide non-stationary seismic traces into several stationary segments adaptively, and then gave a method to estimation Q values. In this work, we shall develop the method of [16].

The compensation methods of attenuation effects (inverse Q filtering) can be applied to make the non-stationary seismic data stationary [17–19]. It is well known that the conventional inverse Q filtering methods can boost noise and introduce numerical instability. To address this issue, many improved strategies have been proposed, one of which is implemented by the inverse problem framework [20]. [20] proposed a seismic absorption compensation method by the inversion scheme in a sparse domain. In this paper, we compensate for the attenuation effects by this inverse framework in the sparse domain.

After compensation, the non-stationary seismic data is turned into the stationary one. In the following parts, we review the super-resolution inversion based on the convolution