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A Fixed Point Approach to the Fuzzy Stability of a Mixed Type Functional Equation

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Abstract: Through the paper, a general solution of a mixed type functional equation in fuzzy Banach space is obtained and by using the fixed point method a generalized Hyers-Ulam-Rassias stability of the mixed type functional equation in fuzzy Banach space is proved.

Key words: mixed functional equation, Hyers-Ulam stability, Fuzzy Banach space, fixed point

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1 Introduction

The stability problem of functional equation originated from a question of Ulam[1] in 1940, concerning the stability of a group hmomorphisms. Heyers[2] gave a first affirmative partial answers to the question of Ulam for Banach spaces. Heyers theorem was generalized by Aoki[3] for additive mapping and by Rassias[4] for linear mappings by considering an unbounded Cauchy difference. A generalization of the Rassias theorem was obtained by Găvruta[5] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias' approach.

The functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y) \tag{1.1}$$

is said to be a quadratic function.

The following cubic functional equation was introduced by Rassias[6]:

$$C(x+2y)+3C(x)=3C(x+y)+C(x-y)+6C(y). \tag{1.2}$$

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The function $f(x) = x^3$ satisfies (1.2), which is called cubic functional equation. And he established the general solution and the generalized Hyers-Ulam-Rassias stability for (1.2).

Later, Gordji *et al.*[7] studied solution and stability of mixed type additive-quadratic-cubic functional equation:

$$f(x+2y) - f(x-2y) = 2[f(x+y) - f(x-y)] + 2f(3y) - 6f(2y) + 6f(y). \tag{1.3}$$

Choonkil[8] gave a fixed point approach to the fuzzy stability of an additive-quadratic-cubic functional equation:

$$\begin{aligned} & f(x+2y) + f(x-2y) \\ = {} & 2[f(x+y) - f(-x-y) + f(x-y) - f(y-x)] \\ & + f(2y) + f(-2y) + 4f(-x) - 2f(x). \end{aligned} \tag{1.4}$$

By using the fixed point methods, the stability problems of several functional equations have been extensively investigated by a number of authors, more reference can be seen in [9]–[10].

In this sequel, we adopt the usual terminology, notations and conventions of the theory in [10].

Definition 1.1 [10] *Let X be a real linear space. A function $N : X \times \mathbf{R} \to [0, 1]$ is said to be fuzzy norm on X, if for all $x, y \in X$ and all $a, b \in \mathbf{R}$,*

(1) $N(x, a) = 0$ *for* $a \le 0$;

(2) $x = 0$ *if and only if* $N(x, a) = 1$ *for* $a > 0$;

(3) $N(ax, b) = N\Big(x, \dfrac{b}{|a|}\Big)$ *if* $a \ne 0$;

(4) $N(x+y, a+b) \ge \min\{N(x,a), N(x,b)\}$;

(5) $N(x, \cdot)$ *is a non-decreasing function on for* $\mathbf{R}$ *and* $\lim\limits_{a\to\infty} N(x, a) = 1$;

(6) *for* $x \ne 0$, *is continuous on* $\mathbf{R}$.

The pair (X, N) is called a fuzzy normed linear space, where X is a linear space and N is a fuzzy norm on X. In the following, we suppose that $N(x, a)$ is left continuous for every x. A sequence $\{x_n\}$ in X is said to be convergent if there exists an $x \in X$ such that $\lim\limits_{n\to\infty} N(x_n - x, t) = 1$ $(t > 0)$. In that case, x is called N-convergent, and denoted by $N - \lim\limits_{n\to\infty} x_n = x$. A sequence $\{x_n\}$ in fuzzy normed space (X, N) is called Cauchy sequence if for each $\varepsilon > 0$ and $\delta > 0$, there exists an $n_0 \in \mathbf{N}$ such that

$$N(x_n - x_m, \delta) > 1 - \varepsilon, \qquad m, n \ge n_0.$$

If each Cauchy sequence is convergent, then the fuzzy norm is said to be complete and the fuzzy normed space is called a fuzzy Banach space.

Let X be a set. A function $d : X \times X \to [0, +\infty]$ is called a generalized metric on X if d satisfies:

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(3) $d(x, z) \le d(x, y) + d(y, z)$.