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SOURCE TERM IDENTIFICATION WITH DISCONTINUOUS DUAL RECIPROCITY APPROXIMATION AND QUASI-NEWTON METHOD FROM BOUNDARY OBSERVATIONS*

El Madkouri Abdessamad and Ellabib Abdellatif[1)]
Faculty of science and technics, Laboratory of applied mathematics and informatics,
University Cadi Ayyad, B.P. 549, Av. Abdelkarim Elkhattabi, Marrakech -40000, Morocco
Email: abdessamad.elmadkouri@edu.uca.ma, a.ellabib@uca.ac.ma

Abstract

This paper deals with discontinuous dual reciprocity boundary element method for solving an inverse source problem. The aim of this work is to determine the source term in elliptic equations for nonhomogenous anisotropic media, where some additional boundary measurements are required. An equivalent formulation to the primary inverse problem is established based on the minimization of a functional cost, where a regularization term is employed to eliminate the oscillations of the noisy data. Moreover, an efficient algorithm is presented and tested for some numerical examples.

Mathematics subject classification: 65N38, 65N21.
Key words: Boundary element method, Inverse source problem, Quasi-Newton methods.

1. Introduction

Inverse problems have gained a very large interest in modern applied mathematics, they arise in a variety of important applications in science and industry, where the aim is to approximate some unknown physical attributes given some measurements related indirectly to this attributes, loosely speaking, we often say an inverse problem is where we measure an effect and want to determine the cause. A practically important class of inverse problems are inverse source problems, such problems appear widely in several branches of engineering disciplines, as examples, crack determination [1, 2], heat source determination [3], inverse heat conduction [4], electromagnetic source identification [5], applications to pollution in the environment [6, 7], dislocation problems [8], biomedical imaging techniques [9, 10], electroencephalography/magnetoencephalography (EEG/MEG) problems [11, 12], the reconstruction of obstacles [42], optical tomography [13], Stefan design problems [14] and the determination of the coefficient of differential equations [30].

A rigorous theoretical background on the investigation of the inverse source problem can be found in [15, 16]. As most inverse problems, the determination of the source term is severely ill-posed problem in the sense of Hadamard [17], namely, any small perturbation in the acquired measurements data generates a colossal error in the solution. A variety of numerical studies were intensively interested in the study of the inverse source problem, which can be divided into two classes: algebraic recovery and projection method, algebraic methods require direct

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1) Corresponding author

algebraic relations between the measurements data and the source parameters while projection algorithms aim to minimize the error between the measured data and the solution of the forward problem, we may refer to [18, 19] and the references therein for more details.

Many approaches were proposed to tackle this problem, some efficient algorithms were stated in the work of Hon et al. [20, 21] to recover the source term in the Poisson equation, in [22] the determination of hidden obstacles via the rational method was investigated, the use of the Green's function method is studied in [20] to recover both the intensities and locations of unknown point sources from scattered boundary measurements. The method of fundamental solutions was successfully applied to solve inverse heat conduction problems in [23, 24]. The application of the dual reciprocity method was firstly applied for the inverse source problem by Kagawa et al. [25] and Trelep et al. [26] where they reconstruct the strength of a source term from boundary observations (only the free error case is studied), then Farcas et al. [27] were interested in recovering the electrical charge density distribution in a two-dimensional electrostatic field where a Tikhonov regularization is employed due to the non uniqueness of the solution of the problem. For an interesting state of the art, the reader may refer to [28, 29] and the references therein, some other interesting related works may be found in [31–34].

The mathematical formulation of 2D problem in the case of a nonhomogenous anisotropic media which occupies an open bounded domain in $\mathbf{R}^2$ with a sufficiently regular boundary Γ such that $\Gamma = \Gamma_D \cup \Gamma_N$, Γ_D, $\Gamma_N \neq \emptyset$ and $\Gamma_D \cap \Gamma_N = \emptyset$ is summarized in the following

$$\begin{cases} -\nabla.(\xi \nabla u) + au = f & \text{in } \Omega, \\ u = g & \text{on } \Gamma_D, \\ (\xi \nabla u) \cdot n = h & \text{on } \Gamma_N, \end{cases} \tag{1.1}$$

where ξ, a, f, g and h are some suitable prescribed functions, $n = (n_x, n_y)$ is the outward unit normal to Γ, in this case, this problem is called direct problem. In this work, we are interested in a reconstruction inverse source problem where the source term f is unknown. The aim in the reconstruction inverse problem is to find the unknown source term f based on the supplementary data provided on the part of the boundary.

Consider the problem where f is unknown and assume that it is possible to measure u on Γ_N. This gives rise to the supplementary condition

$$u = u_d \qquad \text{on } \Gamma_N, \tag{1.2}$$

where u_d is given function. We formulate the inverse source problem as an optimization problem, in order to simulate realistic problems we investigate the case where additional measurements are perturbed, hence a regularization technique is employed. The main objective of this paper is the development of a methodology to estimate the source term with the combination of dual reciprocity approximation and a Quasi-Newton method.

The direct problem is solved using a discontinuous dual reciprocity method (see [35]), whereas the minimization process is solved using the well known Broyden-Fletcher-Goldfarb-Shanno BFGS quasi-Newton algorithm [36–39].

The outline of this paper is structured as follows. The mathematical formulation of the inverse source problem is examined in Section 2, and the third section is devoted to the description of the resolution of the forward problem via the discontinuous dual reciprocity boundary element method Section 4 presents a numerical algorithm for the recovery of the source term is stated, then in Section 5 we assess the efficiency of the proposed algorithm. Finally our conclusions are drawn in the last section.