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Finite-time Stability of Nonlinear Fractional Order Systems with a Constant Delay*

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Abstract In this paper, based on the generalized Gronwall inequality and the method of steps, an approach to the finite-time stability of nonlinear fractional order systems with a constant delay is proposed. A sufficient condition for finite-time stability of considered systems is presented. Compared with the finite-time stability criteria in the existing literature, our results are less conservative. Two examples are given to illustrate the effectiveness of the proposed theorem.

Keywords Finite-time stability, Time delay, Generalized Gronwall inequality.

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1. Introduction

Fractional calculus is playing an increasingly key role in science and engineering fields [19]. Some eminent results have been obtained in fractional differential systems recently [7, 12, 25–27]. One of most important tasks in fractional differential systems is stability analysis. Many researchers focus on the asymptotic stable. However, in practical engineering applications, many systems state trajectories must not exceed a certain bound over a given finite-time interval. The systems of short-time working, for example, missile systems, satellite systems, flight control systems [5], robotic manipulation systems [13], are main examples of such applications.

Finite-time stability of fractional delay differential systems is initially investigated by Lazarevic [8, 9], after that time, many authors [6, 16, 21, 23] adopted the similar approach in [9] to study the fractional delay differential systems. However, the proof of the main theorems in these papers contains a flaw, that is, $f(t) = \sup_{-\tau\leq\theta\leq 0} \|x(t+\theta)\|$ is not always monotone increasing with respect to t, which leads to that $F_1(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f(s)ds$ is not always monotone increasing with respect to t. As a result, the Gronwall inequality can't be used in this

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case. Recently, some different techniques are developed without the help of $f(t) = \sup_{-\tau \le \theta \le 0} \|x(t+\theta)\|$. In 2018, the authors [22] tried to overcome this difficulty by transforming the delayed term $\|x(t-\tau)\|$ to non-delayed term $\|x(t)\|$. Unfortunately, the obtained result is also incorrect, since the proof [22] is based on that $F_2(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} \|u(s)\| ds$ is monotone increasing with respect to t, which is not always correct (see [4]). Similar problem exists in [28, Lemma 2.4].

The main difficulty in using Gronwall inequality to investigate the finite-time stability of nonlinear fractional order systems with time delays is in estimating the delayed term. In [17], Phat et al. overcame this difficulty successfully by constructing a new auxiliary function $u(t) = \sup_{\theta \in [-h,t]} \|x(\theta)\|$. Some different approaches, without using Gronwall inequality to investigate the stability of fractional with time delays, were presented in [10, 11] based on delayed Mittag-Leffler type matrix. In addition, Thanh et al. [20] proposed an approach based on the Laplace transform and a independent-delay Lyapunov functional $V(x(t))$ to study finite-time stability of nonlinear fractional-order systems with interval time-varying delay. In [1–3], the Holder inequality was introduced to guarantee that fractional order delayed system is finite-time stable. The above discussions inspire us for the present investigation.

In this paper, we adopt the method of steps and the generalized Gronwall inequality to give a sufficient condition for the finite-time stability of nonlinear fractional order delay system

$$\begin{cases} {}_0^C D_t^\alpha x(t) = A_0 x(t) + A_1 x(t-\tau) + f(t, x(t), x(t-\tau)), t \in [0, T], \\ x(t) = \varphi(t), t \in [-\tau, 0]. \end{cases} \tag{1.1}$$

The rest of the paper is organized as follows. In Section 2, some definitions and properties of the fractional delay differential equations are introduced. In Section 3, a proof of uniqueness theorem of nonlinear fractional-order time varying delay system as an extension to [21, Theorem 3.2] is given. In Section 4, a gap existing in the proof of [21, Theorem 3.3] is pointed out. Moreover, a new estimate value of the solution of the systems (1.1) is given and a sufficient condition for finite-time stability of the systems (1.1) is presented. In Section 5, two examples are given to illustrate our results.

2. Preliminaries

In this section, we give some basic definitions and notations. Let $\|x(t)\|_1$ be the 1-norm of a vector $x(t) \in R^n$, where $\|x(t)\|_1 = \sum_{i=1}^n |x_i(t)|$. The induced norm $\|A\|_1$ of the matrix A is defined as $\|A\|_1 = \max_{1\le j\le n} \sum_{i=1}^n |a_{i,j}|$. Throughout this paper, all the notation $\|\cdot\|$ means $\|\cdot\|_1$.

Definition 2.1. [18] For $x(t) \in L^1([0, +\infty), R)$, the Riemann-Liouville integral of order α for x is defined by

$${}_0I_t^\alpha x(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} x(s) ds,$$

where $\Gamma(\cdot)$ is the gamma function.