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Bifurcations of Double Homoclinic Loops with Inclination Flip and Nonresonant Eigenvalues

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Abstract In this work, bifurcation analysis near double homoclinic loops with W^s inclination flip of Γ_1 and nonresonant eigenvalues is presented in a four-dimensional system. We establish a Poincaré map by constructing local active coordinates approach in some tubular neighborhood of unperturbed double homoclinic loops. Through studying the bifurcation equations, we obtain the condition that the original double homoclinic loops are persistent, and get the existence or the nonexistence regions of the large 1-homoclinic orbit and the large 1-periodic orbit. At last, an analytical example is given to illustrate our main results.

Keywords Double homoclinic loops, Nonresonant eigenvalues, Inclination flip, Periodic orbit, Bifurcation.

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1. Introduction

During the last few decades, bifurcations of homoclinic or heteroclinic orbits are always widely met in applications, and they have been investigated extensively (see [1–36] and the further references therein). Notably, to the best of the authors' knowledge, only a few concerned the bifurcation of double homoclinic loops. Han and Bi [7] investigated the existence of homoclinic bifurcation curves and small and large limit cycles bifurcated from a double homoclinic loop under multiple parameter perturbations for general planar systems. Han and Chen [8] gave the number of limit cycles near double homoclinic loops under perturbations in planar Hamiltonian systems. Lu [18] obtained codimension 2 bifurcations of twisted double homoclinic loops in higher dimensional systems. Ragazzo [23] investigated the stability of sets that were generalizations of the simple pendulum double homoclinic loop. In our recent work [33, 34], codimension 2 bifurcations of double homoclinic loops and codimension 3 bifurcations of nontwisted double homoclinic loops with resonant eigenvalues were studied.

Bifurcations on inclination flips have been developed in homoclinic or heterclinic loops. Homburg et al. [11] studied three parameter unfolding of resonant homoclinic orbits with orbit flip or inclination flip. Oldeman et al. [22] presented a numerical investigation of the unfolding for a specific three-dimensional vector field, which was

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constructed by Sandstede [24] to explicitly obtain the homoclinic loop with inclination flip and orbit flip. Shui et al. [26] studied codimension 3 nonresonant homoclinic orbit bifurcation with two inclination flips. However, there is no attention to the problem of double homoclinic loops with inclination flips. Motivated by this fact, we will study the problems of homoclinic and periodic orbits bifurcated from double homoclinic loops with W^s inclination flip of Γ_1 and nonresonant eigenvalues in four dimensional systems. Generally speaking, the bifurcation is more complicated as Γ is inclination flip and double homoclinic loops have higher codimension than a single homoclinic loop under the same conditions. Therefore, our work will be more difficult and challenging.

The rest of this paper is organized as follows. In Section 2, some hypotheses are given for our discussion and the normal form is established. In Section 3, the Poincaré map is set up and the bifurcation equations are given. In Section 4, by analysing bifurcation equations, the rich results of inclination flip bifurcations are obtained under different conditions. In Section 5, we give an analytical example to clarify our main results. A brief conclusion ends the paper in Section 6.

2. Hypotheses and Normal form

Consider the following C^r system and its unperturbed system

$$\dot{z} = f(z) + g(z, \nu), \tag{2.1}$$

$$\dot{z} = f(z), \tag{2.2}$$

where r is large enough, $z \in R^4$, $\nu \in R^l$, $l \geq 3$, $0 < |\nu| \ll 1$, $f(0) = 0$, $g(0, \nu) = g(z, 0) = 0$.

We make the following assumptions, which are shown in Figure 1.

(H_1) The linearization $Df(0)$ has simple real eigenvalues at the equilibrium 0: $-\rho_2, -\rho_1, \lambda_1$, λ_2 satisfying

$$-\rho_2 < -\rho_1 < 0 < \lambda_1 < \lambda_2 \qquad \text{and} \qquad \rho_1 > \lambda_1.$$

(H_2) System (1.2) has double homoclinic loops $\Gamma = \Gamma_1 \cup \Gamma_2, \Gamma_i = \{z = r_i(t) : t \in R,\ r_i(\pm\infty) = 0\}$ and $\dim(T_{r_i(t)} W^s \cap T_{r_i(t)} W^u) = 1,\ i = 1, 2$, where W^s and W^u are the stable and unstable manifolds of 0, respectively.

(H_3) Let $e_i^{\pm} = \lim_{t \to \mp\infty} \dfrac{\dot{r}_i(t)}{|\dot{r}_i(t)|}$, and $e_i^+ \in T_0 W^u, e_i^- \in T_0 W^s$ be unit eigenvectors corresponding to λ_1 and $-\rho_1$, respectively, and satisfying $e_1^+ = -e_2^+$, $e_1^- = -e_2^-$.

(H_4) $\mathrm{Span}\{T_{r_i(t)} W^u, T_{r_i(t)} W^s, e_i^+\} = R^4$ as $t \gg 1$,
$\mathrm{Span}\{T_{r_1(t)} W^u, T_{r_1(t)} W^s, e_1^-\} = R^4$ as $t \ll -1$,
$\mathrm{Span}\{T_{r_2(t)} W^u, T_{r_2(t)} W^s, T_{r_1(t)} W^{ss}\} = R^4$ as $t \ll -1$.

The hypotheses (H_4) implies that W^s of Γ_1 is inclination flip. Furthermore, both W^s and W^u of Γ_2 as well as W^u of Γ_1 have the strong inclination property. That is to say, a general vector in $T_{r_2(t)} W^s$ (resp. $T_{r_i(t)} W^u$) not belonging to $\mathrm{span}\{\dot{r}(t)\}$ should go to the strong stable (resp. unstable) direction as $t \to +\infty$ (resp. $t \to -\infty$).