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Revealing the Escape Dynamics in a Hamiltonian System with Five Exits

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Abstract The scope of this work is to reveal, by means of numerical methods, the escape process in a Hamiltonian system with five exits which describes the problem of rearrangement multichannel scattering. For determining the influence of the energy on the nature of the orbits we classify starting conditions of orbits in planes of two dimensions. All the complex basins of escape, associated with the five escape channels of the system, are illustrated by using color-coded diagrams. The distribution of time of the escape is correlated with the corresponding escape basins. The uncertainty (fractal) dimension along with the (boundary) basin entropy are computed for quantifying the degree of fractality of the dynamical system.

Keywords Hamiltonian systems, Escapes, Fractality, Basin entropy.

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1. Introduction

One of the most fascinating topics in nonlinear mechanics and dynamics is, without any doubt, the problem of escapes in dynamical systems (e.g., [1–17]). In these systems a finite energy of escape exists. Consequently, the test particles which are usually launched in the central region of the system find, sooner or later, one of the channels of escape in the zero velocity curve and escape. However, it should be pointed out that the existence of escape channels does not necessarily mean that all orbits must escape to infinity. This is true, taking into account that in many dynamical system bounded orbits exist for which an additional integral of motion prohibit them from escaping. Furthermore, for some orbits the required

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time for escape could be extremely long, in relation to the natural crossing time of the system.

It is well-known that the problem of escapes in dynamical systems is also closely related with the topic of chaotic scattering (e.g., [18]). In this case a test particle, coming outside from the system (usually from infinity), enters the scattering region, where its trajectory is altered, and then either it stays bounded to the system or escapes again to infinity. Over the years, the phenomenon of chaotic scattering has been extensively studied from the point of view of chaos theory (e.g., [19, 20, 22–24, 46]).

In dynamical systems, where escape channels are present, an important issue is to locate the escape basins, associated with the channels of escape (e.g., [3, 25–27]). The escape basins are similar to the attraction basins in dissipative systems (e.g., [28–32]) and also similar to the Newton-Raphson convergence basins (e.g., [33–35]).

For revealing the basins of escape in a dynamical system we have to scan, and therefore classify, starting conditions of orbits in planes of two dimensions. In this work, we will adopt the numerical approach successfully used in several previous papers for determining the basins of escape in simple Hamiltonian system, such as the Hénon-Heiles system (e.g., [28, 36, 37]) but also in more complicated ones, such as the restricted problem of three bodies (e.g., [38, 39]) or open billiards (e.g., [40]) and other applied leaking systems (e.g., [41]). According to this approach, the usual polar coordinates (r, ϕ) will be used for expressing the initial velocities of the orbits, in an attempt to maintain the intrinsic symmetries of the system.

Potential holes over many dimensional configuration spaces with several exit channels play an important role in reactive scattering (e.g., [42, 43]). As a typical process of reactive scattering we can think of a fragment consisting of k particles colliding with another fragment of $N - k$ particles. So we have in total a N particle system. When all particles have attractive interactions among each other, then the total potential has a minimum when all particles are close together and energy is needed to split the whole collection of particles into two or more fragments. In general, there are many possibilities to separate the system into various groups of particles and each one of these possible groupings defines one asymptotic arrangement channel. Each one of these arrangement channels has its own energy threshold where this arrangement starts to exist. For each fixed value of the energy a finite number of arrangement channels are open, i.e. energetically accessible. In our study we only think of scattering processes at moderate values of the total energy, where a description by some effective potential is appropriate. We do not consider events with the production of particle-antiparticle pairs which become relevant at very high energies. We also do not include the production of photons by the scattering process. If some of the particles are of the same type, then the energy is invariant under an exchange of these equal particles. This leads to a corresponding discrete symmetry of the dynamics in the phase space. Also the effective potential will then show this discrete symmetry. Therefore, it is relevant to study scattering potentials with discrete symmetries. For our analysis we will use a simple Hamiltonian with five escape channels, which may represent the effect of reactive scattering.

The article is structured as follows: In Section 2 we describe the mathematical formulation of the Hamiltonian system under consideration. The following Section 3 presents the main numerical outcomes of our investigation, regarding the orbital and escape dynamics of the system. In Section 4 we provide quantitative arguments about fractal degree of the Hamiltonian system, while we also compare the results of