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Geometric Properties and Exact Travelling Wave Solutions for the Generalized Burger-Fisher Equation and the Sharma-Tasso-Olver Equation*

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Abstract In this paper, we study the dynamical behavior and exact parametric representations of the traveling wave solutions for the generalized Burger-Fisher equation and the Sharma-Tasso-Olver equation under different parametric conditions, the exact monotonic and non-monotonic kink wave solutions, two-peak solitary wave solutions, periodic wave solutions, as well as unbounded traveling wave solutions are obtained.

Keywords Monotonic and non-monotonic kink wave, periodic wave, exact solution, integrable system, Burger-Fisher equation, Sharma-Tasso-Olver equation.

MSC(2010) 34A, 35D, 45G.

1. Introduction

It is well known that finding exact solutions of nonlinear wave equations is of great significance because a lot of mathematical models of describing physical phenomena are arising in physics, mechanics, biology, chemistry and engineering. Various powerful methods for obtaining explicit exact traveling wave solutions to nonlinear equations have been developed such as the inverse scattering method, Darboux transformation method, Hirota bilinear method, homogeneous balance method, tanh-function method and so on. For examples, the generalized Burgers-Fisher equation

$$u_t + \alpha u^m u_x + \beta u_{xx} + \gamma u(1 - u^m) = 0 \tag{1.1}$$

has a wide range of applications in plasma physics, fluid mechanics, capillary-gravity waves, nonlinear optics and chemical physics, where α, β and $\gamma \in \mathbf{R}, m$ is positive constant. By using different method, [1,7,8,11,12,14,16] have obtained some exact explicit traveling wave solutions of equation (1.1).

For the following double nonlinear dispersive equation (it was called the Sharma-Tasso-Olver equation):

$$u_t + \alpha(u^3)_x + \frac{3}{2}\alpha(u^2)_{xx} + \alpha u_{xxx} = 0, \tag{1.2}$$

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where α is a real parameter, it was first derived as an example of odd members of the Burgers hierarchy by extending the "linearization" achieved through the Cole-Hopf ansatz to equations containing as highest derivatives odd space derivatives. To find the exact traveling wave solutions of equation (1.2), many physicists and mathematicians have paid much attentions to this equation in recent years due to its importance in mathematical physics (see [?, 3, 4, 6, 9–11, 15], et al.).

Letting $u(x,t) = \phi(x - vt) = \phi(\xi)$, where $\xi = x - vt$ and v stand for the velocity of wave, substituting it into equation (1.1), we have

$$\phi'' + \frac{1}{\beta}(\alpha\phi^m - v)\phi' + \frac{\gamma}{\beta}\phi(1 - \phi^m) = 0, \tag{1.3}$$

where "$'$" stand for the derivative with respect to ξ. Without loss of generality, by a parameter transformation, we can take $\beta = 1$.

Equation (1.3) is equivalent to the planar cubic system:

$$\frac{d\phi}{d\xi} = y, \quad \frac{dy}{d\xi} = -(\alpha\phi^m - v)y - \gamma\phi(1 - \phi^m). \tag{1.4}$$

Letting $u(x,t) = \phi(x - vt) = \phi(\xi)$, substituting it into equation (1.2) and integrating the obtained equation once, taking the integral constant as 0, we obtain

$$\phi'' + 3\phi\phi' - \frac{v}{\alpha}\phi + \phi^3 = 0. \tag{1.5}$$

Equation (1.5) is equivalent to the planar cubic system:

$$\frac{d\phi}{d\xi} = y, \quad \frac{dy}{d\xi} = -3\phi y + \frac{v}{\alpha}\phi - \phi^3. \tag{1.6}$$

To the best our knowledge, we notice that the dynamical behavior of travelling wave solutions of systems (1.4) and (1.6) have not be studied before. From the view point of the theory of dynamical systems, we hope to know which orbit corresponds to a known exact solution. In other words, we need to understand the geometric properties of all known exact solutions. In this paper, under some integrable parameter conditions, we consider two traveling wave systems of equation (1.1) and equation (1.2) and answer the above problem. The dynamics and exact parametric representations for the traveling wave solutions of equations (1.1) and (1.2) can be given.

This paper is organized as follows. In section 2, we consider the phase portraits of system (1.4) in the integrable cases and give exact kink wave solutions for the generalized Burgers-Fisher equation (1.1). In section 3, we discuss the phase portraits and the graphs of level curves (i.e. the orbits of (1.4) for any fixed h) defined by $H(\phi, y) = h$ of system (1.6) for $v\alpha > 0, v = 0$ and $v\alpha < 0$, respectively. In section 4, we discuss the dynamical behavior of solutions of system (1.4) and figure out exact explicit parametric representations of the traveling wave solutions of the Sharma-Tasso-Olver equation (1.2).