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Nonexistence of Nonconstant Positive Steady States of a Diffusive Predator-prey Model with Fear Effect

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Abstract In this paper, we investigate a diffusive predator-prey model with fear effect. It is shown that, for the linear predator functional response case, the positive constant steady state is globally asymptotically stable if it exists. On the other hand, for the Holling type II predator functional response case, it is proved that there exist no nonconstant positive steady states for large conversion rate. Our results limit the parameters range where complex spatiotemporal pattern formation can occur.

Keywords Reaction-diffusion, fear effect, flobal stability, nonexistence of nonconstant steady states.

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1. Introduction

The interaction between predator and prey is one of fundamental ecological phenomena. Adding the random movement in the spatial habitat, reaction-diffusion systems have been used to described the interaction and dispersal of the predator and prey species [1, 6, 7, 15, 17, 18, 21, 22]. Recently some researchers found that the fear of the predators could lead to the reduction of the prey, see [8–10, 19, 23] and references therein. A reaction-diffusion predator-prey system with fear effect and predator-taxis is proposed in [20]:

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1\Delta u + \alpha\nabla\cdot(\beta(u)u\nabla v) + \dfrac{ru}{1+kv} - du - au^2 - \dfrac{buv}{1+qu}, & x\in\Omega,\ t>0,\\ \dfrac{\partial v}{\partial t} = d_2\Delta v - m_1 v - m_2 v^2 + \dfrac{cuv}{1+qu}, & x\in\Omega,\ t>0,\\ \partial_n u = \partial_n v = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0) = u_0(x) \ge (\not\equiv) 0,\ v(x,0) = v_0(x) \ge (\not\equiv) 0, & \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ are the density functions of the prey and predator population; Ω is a bounded domain in $\mathbb{R}^N (N \le 3)$ with a smooth boundary $\partial\Omega$; d_1

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and d_2 are the diffusion coefficients of the prey and predator respectively, and $\alpha\nabla\cdot(\beta(u)u\nabla v)$ represent the predator-taxis that predator moves toward high prey concentration location; $m_1 > 0$ and $m_2 \geq 0$ account for the death rate and crowding effect of the predator, $r > 0$ and $d > 0$ are the birth and death rates of the prey respectively, and $a > 0$ reflects the intro-species competition of the prey; $b > 0$ and $c > 0$ measure the interaction strength between the predator and prey; $q \geq 0$ measures the prey's ability to evade attack and $u/(1 + qu)$ is the Holling type II functional response; and $k > 0$ represents the fear effect. For the corresponding kinetic model, it is known that high levels of fear can stabilize the positive steady state, and low levels of fear can induce multiple limit cycles leading to bistable phenomenon [19]. For the diffusive model (1.1) with $q = 0$, it is shown that the unique positive constant steady state is globally asymptotically stable under certain conditions, and for $q \neq 0$, complex spatiotemporal pattern formation can occur [20].

In this paper, we revisit model (1.1) without considering the predator-taxis, that is,

$$\begin{cases} \dfrac{\partial u}{\partial t} = d_1\Delta u + \dfrac{ru}{1+kv} - du - au^2 - \dfrac{buv}{1+qu}, & x\in\Omega,\ t>0,\\ \dfrac{\partial v}{\partial t} = d_2\Delta v - m_1 v - m_2 v^2 + \dfrac{cuv}{1+qu}, & x\in\Omega,\ t>0,\\ \partial_n u = \partial_n v = 0, & x\in\partial\Omega,\ t>0,\\ u(x,0) = u_0(x) \geq (\not\equiv) 0,\ v(x,0) = v_0(x) \geq (\not\equiv) 0. \end{cases} \tag{1.2}$$

We find that, for $q = 0$ (Lotka-Volterra case), the positive constant steady state is globally asymptotically stable if exists, and for $q \neq 0$ (Holling Type II case), there exists no nonconstant positive steady states with large conversion rate c. Our result for global stability is proved under weaker condition that the ones in [20] but also without predator-taxis. Our results give some ranges for the model parameters within which, spatiotemporal pattern formation cannot occur, and supplement some results obtained in [20].

The model (1.2) is a variant of more commonly studied Rosenzweig-MacArthur predator-prey model with Holling type II functional response [13, 18, 22]. By using conversion rate c as a variable parameter, they showed the existence of Hopf and steady state bifurcations, and there exist no nonconstant steady states when c is large or small, which implies that the global bifurcating branches of steady state solutions of system are bounded loops. Related results were also obtained for the diffusive predator-prey model with Holling type III predator functional response [2, 16], or other more general functional responses [4], or other growth functions [5], or delay effect [3].

The rest of the paper is organized as follows. In Section 2, we consider the global stability of the constant positive steady state for the Lotka-Volterra predation case. In Section 3, we show the nonexistence of nonconstant positive steady states for the Holling type II predation case.

2. The Lotka-Volterra case

In this section, we show that, when $q = 0$, the constant positive steady state of model (1.2) is globally asymptotically stable if it exists. Therefore, complex pattern formation cannot occur. Clearly, for $q = 0$, model (1.2) has a constant positive steady state (u_*, v_*) if and only if $r > d + \frac{am_1}{c}$, see [20]. Then we have: