Communications
in
Mathematical
Research
27(4)(2011), 343–348

Sharp Weighted Estimates for a Class of n-dimensional Hardy-Steklov Operators*

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Communicated by Ji You-qing

Abstract: In this paper, we study one class of n-dimensional Hardy-Steklov operators which has important applications in the technical analysis in equity markets. We establish their weighted boundedness and the corresponding operator norms on both $L^p(\mathbf{R}^n)$ and $\mathrm{BMO}(\mathbf{R}^n)$.

Key words: Hardy-Littlewood average, Hardy-Steklov operator, BMO
2000 MR subject classification: 42B25, 42B99
Document code: A
Article ID: 1674-5647(2011)04-0343-06

1 Introduction

Let a function $w(x) \geq 0$, $x \in [a, b]$ with $0 < a < b$, be given. Then for a measurable function $f : \mathbf{R}^n \to \mathbf{C}$, the n-dimensional weighted Hardy-Steklov operators which we study are defined as

$$(T_{w([a,b])}f)(x) = \frac{1}{(b-a)x}\int_{ax}^{bx} f(t)w(t/x)\mathrm{d}t \tag{1.1}$$

and

$$(\widetilde{T}_{w([a,b])}f)(x) = \frac{1}{bx-ax}\int_{x/a}^{x/b} f(t)(x/t)w(x/t)\mathrm{d}t. \tag{1.2}$$

When $n = 1$, (1.1) and (1.2) become the classical Hardy-Steklov operators (HSO).

Definition 1.1[1] *We call an operator*

$$(Hf)(x) = \int_{a(x)}^{b(x)} f(t)\mathrm{d}t, \qquad f(t) \geq 0,\ 0 < t < \infty$$

an HSO if the functions $a(x)$ and $b(x)$ satisfy

$$\begin{aligned} a(x) &= b(x) = x, && x = 0 \text{ or } x = \infty; \\ a(x) &< b(x), && 0 < x < \infty. \end{aligned}$$

*Received date: Sept. 19, 2010.
Foundation item: The NSF (10901076, 10931001) of China.

The corresponding moving averaging operator of HSO is defined by

$$(\widehat{H}f)(x) = \frac{1}{(b(x)-a(x))}\int_{a(x)}^{b(x)} f(t)\mathrm{d}t, \qquad f \geq 0.$$

This operator in its various forms is of considerable importance to the technical analysts in the study of equity markets. These technical analysts try to predict the future of the stock price or the future of an equity market solely on the base of the past performance of the stock price or market valuation, respectively.

The study of $T_{w([a,b])}$ and $\widetilde{T}_{w([a,b])}$ seems to be of interest as it is related closely to the Hardy-Littlewood maximal operators in harmonic analysis (see [2]) and technical analysis in the study of equlity markets (see [1]). For example, if we make the change of variable $t' = t/x$, then (1.1) and (1.2) become

$$(T_{w([a,b])}f)(x) = \frac{1}{(b-a)}\int_a^b f(tx)w(t)\mathrm{d}t \tag{1.3}$$

and

$$(\widetilde{T}_{w([a,b])}f)(x) = \frac{1}{b-a}\int_a^b f(x/t)t^{-n}w(t)\mathrm{d}t, \tag{1.4}$$

respectively. Note that in the following analysis we only consider (1.3) and (1.4) instead of (1.1) and (1.2). If $w \equiv 1$ and $[a,b] = [0,1]$, then $T_{w([a,b])}$ and $\widetilde{T}_{w([a,b])}$ are just reduced to the classical Hardy-Littlewood average T and the classical Cesáro operator $\widetilde{T}$ (see [3]):

$$Tf(x) = \frac{1}{x}\int_0^x f(y)\mathrm{d}y, \qquad x \neq 0,$$

$$\widetilde{T}f(x) = \begin{cases} \displaystyle\int_x^\infty \frac{f(y)}{y}\mathrm{d}y, & x > 0; \\ \displaystyle -\int_{-\infty}^x \frac{f(y)}{y}\mathrm{d}y, & x < 0. \end{cases}$$

In addition, $T + \widetilde{T}$ becomes the Calderón maximal operator

$$(Tf)(x) + (\widetilde{T}f)(x) = \frac{1}{x}\int_0^x f(y)\mathrm{d}y + \int_x^\infty \frac{f(y)}{y}\mathrm{d}y, \qquad x > 0. \tag{1.5}$$

Many works have been done about Hardy-Littlewood operators and its commutators which we refer readers to [4]–[7].

Definition 1.2[8] *A locally integrable function b is said to be in* BMO($\mathbf{R}^n$) *if for any cube* $Q \subset \mathbf{R}^n$

$$\|b\|_{\mathrm{BMO}(\mathbf{R}^n)} = \sup_Q \frac{1}{|Q|}\int_Q |b(x) - b_Q|\mathrm{d}x < \infty,$$

where

$$b_Q = \frac{1}{|Q|}\int_Q b(x)\mathrm{d}x.$$

In what follows, for each number $\lambda > 0$ and cube $Q \subset \mathbf{R}^n$, let λQ be the cube whose measure has $\lambda|Q|$, and whose center is the same as that of Q. For the standard work on BMO($\mathbf{R}^n$), we refer the reader to [6], [7], [9] and [10].

Inspired by the work in [1], [4] and [7], we study the norm estimates of $T_{w([a,b])}$ and $\widetilde{T}_{w([a,b])}$ in Section 2.