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Cyclic Group Action of Composite Order on Indefinite 4-manifolds*

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Abstract: In this paper, we study the possibilities for several kinds of topological, locally linear cyclic group actions of non-prime order on some closed, simply connected 4-manifolds with indefinite intersection form. Especially, we discuss the existence of locally linear pseudofree C_9 action on this kind of 4-manifolds.

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1 Introduction

In the earlier study, Edmonds[1] gave a collection of various consequences of the existence of finite cyclic group actions on simply connected topological 4-manifold. Especially, he obtained the nonexistence of locally linear involutions on certain 4-manifolds with positive definite intersection form. Edmonds[2] explored the group actions on the E_8 4-manifolds. Based on these results, Edmonds[3] studied the existence of locally linear cyclic group action with composite order on simply connected 4-manifold with positive definite intersection form. The purpose of this paper is to explore the possibilities for several kinds of topological, locally linear cyclic group actions of non-prime order on some closed, simply connected 4-manifolds with indefinite intersection form.

The paper is organized as follows. In Section 2, we give some preliminaries such as transformation groups, the Lefschetz fixed point formula, the G-signature formula and some basic lemmas in number theory. In Section 3, we prove the main results of this paper.

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2 Notations and Preliminaries

Let Σ_n be a permutation group, and

$$\sigma = (p_1)(p_2)\cdots(p_r)$$

a conjugacy class of Σ_n, where

$$p_i \geq 1, \qquad n = p^1 + ... + p^r.$$

Let

$$m = \mathrm{lcm}\{p_1, \cdots, p_r\},$$

where lcm means the least common multiple. It is easy to see

$$\sigma^m = \mathrm{Id},$$

so σ generates a cyclic group C_m with order m. Such an action of C_m on the set $\{1, 2, \cdots, n\}$ determines a permutation representation of C_m on $\mathbf{Z}^n$ which we describe as $(p_1) + (p_2) + \cdots + (p_r)$ in the following text.

Let G be a locally linear action on 4-manifold X. Then for each point $x \in X$ there is a neighborhood on which the action of G_x is equivalent to a linear action on some euclidean space, where G_x denotes the isotropy group of x. Besides, if $G_x \approx C_m$, where m is odd, then the fixed point $x \in X$ has a local representation type which can be described by a pair of nonzero ordered integers (a, b). If we fix a generator $g \in G_x$, then the action of g is $(z, w) \to (\zeta^a z, \zeta^b w)$, where

$$\zeta = \exp\{2\pi \mathrm{i}/m\}.$$

A group action is said to be pseudofree if each nontrivial group element has a discrete fixed point set.

For the general theory about transformation groups we refer to [4] and [5].

Let $g : X \to X$ generate an action of C_m on a closed, simply connected 4-manifold X. By the Local Smith theory, $\mathrm{F} = \mathrm{Fix}(g)$ consists of isolated points and surfaces. Besides, by Proposition 2.4 in [1], all fixed surfaces are 2-spheres if and only if the representation g_* on $H_2(X)$ is a permutation representation. In any case, we have the Lefschetz fixed point formula

$$\chi(F) = \Lambda(g) = 2 + \mathrm{Trace}[g_* : H_2(X) \to H_2(X)].$$

We can refer to [5] for details.

Next we assume that the representation g_* on $H_2(X)$ is a permutation representation and g has isolated fixed points x_i and fixed surfaces S_j which are all 2- spheres. Suppose that x_i has local fixed point data (a_i, b_i), S_j has normal Euler number n_j and normal rotation angle data e_j. Then we have the following G-signature formula:

$$\sigma(g, X) = \sum_i \frac{(\zeta^{a_i} + 1)}{(\zeta^{a_i} - 1)} \frac{(\zeta^{b_i} + 1)}{(\zeta^{b_i} - 1)} - \sum_j \frac{4n_j \zeta^{e_j}}{(\zeta^{e_j} - 1)^2}$$

(see [6]).

We introduce some useful lemmas in number theory which are used in proving the main results. We can refer to any elementary number theory book for detail, for example, [7].