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Global Attracting Sets of Neutral Stochastic Functional Differential Equations Driven by Poisson Jumps

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Abstract. By means of the Banach fixed point principle, we establish some sufficient conditions ensuring the existence of the global attracting sets and the exponential decay in the mean square of mild solutions for a class of neutral stochastic functional differential equations by Poisson jumps. An example is presented to illustrate the effectiveness of the obtained result.

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1 Introduction

Attracting sets of stochastic dynamical systems have attracted the increasing attention over the last a few decades due to weaken the stability conditions of stochastic system. Many different arguments have been developed to establish some sufficient conditions ensuring the existence of the global attracting sets. Among others, the (delay) integral inequalities introduced in [1] has been efficiently applied, see, e.g., Wang and Li [2] obtained the global attracting sets of impulsive stochastic partial differential equations with infinite delays by establishing some impulsive-integral inequalities; Long et al. [3] considered the global attracting set and stability of stochastic neutral partial functional

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differential equations with impulses; Liu and Li [4] gave the global attracting sets and the conditions of exponential stability of neutral stochastic partial functional differential equations driven by α-stable processes; Li [5] and Xu and Luo [6] studied the global attracting sets and the conditions of exponential stability of mild solutions to a class of neutral stochastic functional differential equations driven by fBm with Hurst parameter $1/2<\hbar<1$ and $0<\hbar<1/2$, respectively.

On the other hand, stochastic differential equations driven by Poisson jump processes have attracted a great deal of attention. For example, Knoche [7] proved the existence and uniqueness of mild solution for stochastic evolution equations with Poisson jump processes; Röckner and Zhang [8] studied the existence, uniqueness and large deviation principle for stochastic evolution equations driven by Poisson jump processes; Luo and Taniguchi [9] investigated the existence and uniqueness for non-Lipschitz stochastic neutral delay evolution equations driven by Poisson jumps; Hou et al. [10] considered the stability of energy solutions for SPDEs with variable delay and Poisson jump processes; Cui at al. [11] studied the exponential stability for neutral stochastic evolution equations driven by Poisson jumps; Budhiraja et al. [12] obtained the large deviations for stochastic partial differential equations driven by a Poisson random measure.

But, as far as we know that there is no paper which investigates the global attracting sets for stochastic differential equations driven by Poisson jump processes. In order to fill this gap, in this paper, we will investigate by employing the Banach fixed point principle, without requiring to any integral inequality, the existence of the global attracting sets of the mild solution for a class of neutral stochastic functional differential equation driven by Poisson jump processes.

The rest of this paper is organized as follows. In Section 2, we introduce some necessary notations and preliminaries. In Section 3, we state and prove our main results.

2 Preliminary

Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathbb{P})$ be a filtered complete probability space satisfying the usual condition, which means that the filtration is a right continuous increasing family and $\mathcal{F}_0$ contains all P-null sets. Let $(H,\langle\cdot,\rangle,\|\cdot\|)$ and $(K,\langle\cdot,\rangle_K,\|\cdot\|_K)$ be two real, separable Hilbert spaces and $\mathcal{L}(K,H)$ be the space of bounded linear operator from K to H.

Suppose $\{p(t),t\geq 0\}$ is a σ-finite stationary $\mathcal{F}_t$-adapted Poisson point process which takes values in a measurable space $(U,\mathfrak{B}(U))$. The random measure N_p defined by $N_p((0,t]\times A):=\sum_{s\in(0,t]}\mathcal{I}_A(p(s))$ for $A\in\mathfrak{B}(U)$ is called the Poisson random measure induced by $p(\cdot)$. Then, we can define the compensated Poisson random measure $\widetilde{N}$ by $\widetilde{N}(dt,dy)=N_p(dt,dy)-v(dy)dt$, where v is the characteristic measure of N_p. Let $W=W(t)_{t\geq 0}$ be a K-valued Wiener process which is independent of the Poisson point process, defined on $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathbb{P})$ with covariance operator Q, that is

$$\mathbb{E}\langle W(t),x\rangle_K\langle W(s),y\rangle_K=(t\wedge s)\langle Qx,y\rangle_K,\quad \forall x,y\in K,$$