J. Math. Study
doi: 10.4208/jms.v54n4.21.07

Vol. **54**, No. 4, pp. 427-434
December 2021

Domination in Generalized Cayley Graph of Commutative Rings

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Received 21 July 2020; Accepted (in revised version) 6 August 2020
Published online 29 March 2021.

Abstract. Let R be a commutative ring with identity and n be a natural number. The generalized Cayley graph of R, denoted by Γ_R^n, is the graph whose vertex set is $R^n\setminus\{0\}$ and two distinct vertices X and Y are adjacent if and only if there exists an $n\times n$ lower triangular matrix A over R whose entries on the main diagonal are non-zero such that $AX^T=Y^T$ or $AY^T=X^T$, where for a matrix B, B^T is the matrix transpose of B. In this paper, we give some basic properties of Γ_R^n and we determine the domination parameters of Γ_R^n.

AMS subject classifications: 13A99, 05C69

Key words: Ring, Cayley graph, generalized Cayley graph, domination number.

1 Introduction

The theory of Cayley graphs has grown into a substantial branch of algebraic graph theory in the last few decades. The concept of Cayley graph was introduced by Aurther Cayley in 1878 to explain the abstract groups which are described by a set of generators. Cayley graphs of groups have been extensively studied and some interesting results have been obtained (see [3]). Also, the Cayley graphs of semi groups have been considered by some authors (see [5, 8–14, 16]).

Let R be a commutative ring with identity. Sharma and Bhatwadekar [18] defined the comaximal graph on R, denoted by $\Gamma(R)$, with all elements of R being the vertices of $\Gamma(R)$, where two distinct vertices a and b are adjacent if and only if $aR+bR=R$. In [15,19], the authors considered a subgraph $\Gamma_2(R)$ of $\Gamma(R)$ consisting of non unit elements of R,

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and investigated several properties of the comaximal graph. Also the comaximal graph of a non-commutative ring was defined and investigated in [20]. Note that the subgraph of the undirected Cayley graph $\overline{Cay}(R^*,R^*)$ consisting of non-unit vertices is a subgraph of the complement of the comaximal graph $\Gamma_2(R)$. Moreover, the two subgraphs of the undirected Cayley graph $\overline{Cay}(R^*,R^*)$ and the comaximal graph $\Gamma(R)$ consisting of unit elements of R are isomorphic. In view of this, Khashyarmanesh et al. [1] introduced and characterized the rings in terms of the genus and crosscap numbers of the generalized Cayley graph Γ_R^n of a commutative ring R. For a natural number n and a commutative ring R with identity element, we associate a simple graph, denoted by Γ_R^n, with $R^n\setminus\{0\}$ as the vertex set and two distinct vertices X and Y are adjacent if and only if there exists an $n\times n$ lower triangular matrix A over R whose entries on the main diagonal are non-zero such that $AX^T=Y^T$ or $AY^T=X^T$, where for a matrix B, B^T is the matrix transpose of B. In case $n=1$, the resulting graph is the undirected graph $\overline{Cay}(R^*,R^*)$. They determined the clique number of the graph which is always greater than or equal to $|U(R)||R|^{n-1}$, where $U(R)$ is the set of all units in R. In this paper, we obtain the domination number of generalized Cayley graph associated with rings.

Let G be a graph with vertex set V. A subset D of V is called a dominating set of G if every vertex in $V\setminus D$ is adjacent to at least one vertex in D. The domination number of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G. A subset D of V is called total dominating set of G if every vertex in V is adjacent to at least one vertex in D. The total domination number of G, denoted by $\gamma_c(G)$, is the minimum cardinality of a total dominating set of G. A dominating set D is called a connected dominating set if the induced subgraph $\langle D\rangle$ is connected. The connected domination number $\gamma_c(G)$ of a graph G equals the minimum cardinality of a connected dominating set in G. A dominating set D is called an independent dominating set of G if no two vertices of D are adjacent in G. The minimum cardinality of an independent dominating set of G is the independent domination number $\gamma_i(G)$. For basic definitions on graphs, we refer to [4,6,17].

A ring $(R,\mathfrak{m})$ is called local if it has a unique maximal ideal $\mathfrak{m}$. We note that $J(R)$ is the Jacobson radical of R and $Reg(R)$ is the set of regular elements of R. For any set X, let X^* denote the non-zero elements of X. We denote the ring of integers modulo n by $\mathbb{Z}_n$, the field with q elements by $\mathbb{F}_q$. For basic definitions on rings, one may refer [2,7]. Throughout this paper, we assume that R is a commutative ring with identity and $n>1$.

The following results are useful in the subsequent sections.

Lemma 1.1 ([1]). *Let $X=(x_1,x_2,\cdots,x_n)\in R^n\setminus\{0\}$ be a vertex whose first component is a unit and $Y=(y_1,y_2,\cdots,y_n)\in R^n\setminus\{0\}$ be a vertex whose first component is non-zero. Then X and Y are adjacent in Γ_R^n. Further, the induced subgraph of all vertices whose first components are units is a complete graph.*

Theorem 1.1 ([1]). *If R is an integral domain, then Γ_R^n is disconnected. Moreover, Γ_R^n has n components and every component is a refinement of a star. Hence, if R is a field, then Γ_R^n has n complete components.*