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An Averaging Principle for Caputo Fractional Stochastic Differential Equations with Compensated Poisson Random Measure

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Abstract. This article deals with an averaging principle for Caputo fractional stochastic differential equations with compensated Poisson random measure. The main contribution of this article is impose some new averaging conditions to deal with the averaging principle for Caputo fractional stochastic differential equations. Under these conditions, the solution to a Caputo fractional stochastic differential system can be approximated by that of a corresponding averaging equation in the sense of mean square.

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1 Introduction

Most systems in science and industry are perturbed by some random environmental effects, described by stochastic differential equations with (fractional) Brownian motion, Lévy process, Poisson process and etc. A series of useful theories and methods have been proposed to explore stochastic differential equations, such as invariant manifolds , averaging principle , homogenization principle. All of these theories and methods develop to extract an effective dynamics from these stochastic differential equations, which

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is more effective for analysis and simulation. For averaging principle, its often used to approximate dynamical systems with random fluctuations, and provides a powerful tool for simplifying nonlinear dynamical systems. The essence of averaging principle is to establish an approximation theorem that a simplified stochastic differential equation is presented to replace the original one in some senses.

The averaging principle for stochastic differential equations was first introduced by Khasminskii in paper [1], which extending the deterministic result of [2]. Since then, the theory of averaging principle for stochastic differential equations driven by different noise are considered by many authors, see [3–7] .

Because the non-local property of time derivatives, the model of Caputo fractional stochastic differential equations applied in many areas, such as biology, physics and chemistry and etc.. Existence and uniqueness of solution for Caputo fractional stochastic differential have been discussed by many papers. Quite recently, some types of Caputo fractional stochastic (partial) differential equations problem are considered from the dynamic viewpoint. For example, in paper [8], existence of stable manifolds is established. In [9], the existence of global forward attracting set for stochastic lattice systems with a Caputo fractional time derivative in the weak mean-square topology is considered. The asymptotic distance between two distinct solutions under a temporally weighted norm is discussed by [10]. To the best of our knowledge, averaging principle for Captuo fractional differential equations only considered by two articles in present, see [11] and [12], it is worth noting that the average conditions given in these two papers are different, under this conditions, the corresponding averaging conclusions are drawn respectively. In this paper, we also impose a new averaging condition for our framework, which let us to derive the averaging principle for our consider problem from the theoretical derivation.

Let $(\Omega,\mathcal{F},\mathcal{F}_t,\mathbb{P})$ be a complete stochastic base. Let $(Z,\mathcal{B}(Z))$ be a measurable space and $\nu(dz)$ a $\sigma-$finite measure on it. Let $p=(p(t)),t\in D_p$, be a stationary $\mathcal{F}_t-$Poisson point process on Z with characteristic measure ν. The counting measure associated with $p(t)$ is given by, for $A\in\mathcal{B}(Z)$, $N((0,t],A):=\sharp\{t\in D_p: 0<s\leq t, p(s)\in A\}$. Assume that $b:\mathbb{R}_+\times\mathbb{R}^d\to\mathbb{R}^d,\sigma:\mathbb{R}_+\times\mathbb{R}^d\to\mathbb{R}^d\times\mathbb{R}^m$ and $F:\mathbb{R}_+\times\mathbb{R}^d\times Z\to\mathbb{R}^d$ are measurable functions. For $\alpha\in(\frac{1}{2},1)$, we consider a stochastic fractional differential equation with compensated Poisson random measure of the form:

$$D_t^\alpha X_\epsilon(t)=\epsilon b(t,X_\epsilon(t-))\mathrm{d}t+\sqrt{\epsilon}\sigma(t,X_\epsilon(t-))\mathrm{d}B_s+\sqrt{\epsilon}\int_Z F(t,X_\epsilon(t-),z)\tilde{N}(\mathrm{d}t,\mathrm{d}z), \tag{1.1}$$

where ϵ is a small positive parameter, $\tilde{N}(\mathrm{d}t,\mathrm{d}z):=N(\mathrm{d}t,\mathrm{d}z)-\mathrm{d}t v(\mathrm{d}z)$ is the compensated Poisson martingale measures corresponding to $N(\mathrm{d}t,\mathrm{d}z)$ and $\{B_t\}_{t\geq 0}$ is an m-dimensional standard $\mathcal{F}_t$-adapted Brownian motion. We note that the above equation is a classical equation if $\alpha=1$, which has been studied by many authors. Under some conditions, which can be compared with the classic case as in [12], we derive an averaging principle for the stochastic fractional differential system (1.1).

This article is organized as follows. In Section 2 we will give some assumptions and basic results for our theory. The solution of convergence in mean square between the