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AN IMPROVED TWO-GRID TECHNIQUE FOR THE NONLINEAR TIME-FRACTIONAL PARABOLIC EQUATION BASED ON THE BLOCK-CENTERED FINITE DIFFERENCE METHOD*

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Abstract

A combined scheme of the improved two-grid technique with the block-centered finite difference method is constructed and analyzed to solve the nonlinear time-fractional parabolic equation. This method is considered where the nonlinear problem is solved only on a coarse grid of size H and two linear problems based on the coarse-grid solutions and one Newton iteration is considered on a fine grid of size h. We provide the rigorous error estimate, which demonstrates that our scheme converges with order $\mathcal{O}(\Delta t^{2-\alpha}+h^2+H^4)$ on non-uniform rectangular grid. This result indicates that the improved two-grid method can obtain asymptotically optimal approximation as long as the mesh sizes satisfy $h=\mathcal{O}(H^2)$. Finally, numerical tests confirm the theoretical results of the presented method.

Mathematics subject classification: 26A33, 65M06, 65M12, 65M15, 65M55.
Key words: Improved two-grid, Time-fractional parabolic equation, Nonlinear, Error estimates, Numerical experiments.

1. Introduction

Nowadays, fractional differential equations have been applied to describe quite a few physical phenomena [12,13,15–17,22], such as rheology, damping laws, diffusion processes, etc. The time fractional parabolic equations have appeared to describe the transport dynamics in case that Gaussian statistics are no longer followed and the Fick's second law fails to describe the related transport behaviors.

Many research works based on the theoretical and numerical simulation for the time fractional parabolic equations have been studied extensively by many researchers. Xu et al. [12,13] con-

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structed and analyzed stable high order schemes to solve the time-fractional diffusion equation. Zheng et al. [22] studied the space-time spectral method for the time fractional Fokker-Planck initial-boundary value problem. Gao et al. [7] have researched the high order approximation for the time multi-term and distributed-order fractional sub-diffusion equations by finding a special point for the interpolation approximation of the linear combination of multi-term fractional derivatives. Recently, Li and Rui [11] presented two H^1-Galerkin mixed finite element schemes combined with higher accurate interpolation approximation for the distributed order fractional sub-diffusion equations.

The two-grid discretization method has been well developed in recent years. It was first introduced by Xu [20,21] for nonsymmetric indefinite and nonlinear problems. Huang and Chen [8] proposed the multilevel iterative technique for solving the finite element equations of nonlinear singular two-point boundary value problems at almost the same time. The technique was then applied to other problems by many researchers due to its high efficiency. Dawson et al. [4] applied the two-grid technique to the finite difference method for nonlinear parabolic equations. Chen and Liu [1] derived the rigorous convergence estimates of the two-grid finite volume element methods for semilinear parabolic problems. The accelerated two-grid finite element method for solving numerically the two-dimensional Steklov eigenvalue problem was constructed by Weng et al. [19]. Chen et al. [2,3] developed the two-grid mixed finite element method for the nonlinear problems. A two-grid block-centered finite difference method is proposed for solving the Darcy-Forchheimer model describing non-Darcy flow in porous media by Rui and Liu [18]. The other two-grid block-centered finite difference methods for nonlinear non-fickian flow and time fractional equations were developed in [9, 10].

In this paper, we construct a combined scheme of the improved two-grid technique with the block-centered finite difference method. Compared with our previous work in [10], the main contribution is that we can obtain that the convergence rates of velocity and pressure are both $\mathcal{O}(\Delta t^{2-\alpha}+h^2+H^4)$ in different discrete norms by constructing the improved three-step two-grid approach. This algorithm involves three steps: In step 1 we solve a small nonlinear problem on the coarse grid with mesh size H. The linear algebraic system based on the coarse-grid solutions is considered on the fine grid with mesh size h in step 2, which is quite standard in most two-grid approach. In step 3, we solve the corresponding linear system again on the fine-grid by one Newton iteration using the known fine-grid solution obtained in step 2. We should point out that step 3 is a crucial and effective procedure to arrive at $\mathcal{O}(\Delta t^{2-\alpha}+h^2+H^4)$ accuracy for the velocity and pressure. Error estimates are established on non-uniform rectangular grid, which show that the discrete $L^\infty(L^2)$ and $L^2(H^1)$ errors are $\mathcal{O}(\Delta t^{2-\alpha}+h^2+H^4)$ for the velocity and pressure. To the best of the authors' knowledge, this is a first such result for the block-centered finite difference method for fractional equation. Finally, some numerical experiments are presented to show the efficiency of the two-grid method and verify that the convergence rates are in agreement with the theoretical analysis. Moreover, we also give the numerical examples of the nonlinear implicit scheme to illustrate the efficiency of the two-grid block-centered finite difference method.

The paper is organized as follows. In Section 2, we give the governing problem. In Section 3, we present the improved two-grid block-centered finite difference algorithm. Then error estimates for the presented methods are analyzed in Section 4. Finally, some numerical experiments using the block-centered finite difference schemes are carried out in Section 5.

Throughout the paper we use C, with or without subscript, to denote a positive constant, which could have different values at different appearances.