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Partial Eigenstructure Assignment Using Incomplete Measured Modal Data

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Abstract. A partial eigenstructure assignment problem with incomplete measured modal data is considered and the problem of its solvability is discussed. Besides, we propose a method to solve the problem such that unsuitable eigenvalues are replaced by the required ones while the remaining eigenstructure stays unchanged. The method is easily implementable and does not use QR factorization and singular-value decomposition. We also apply this method in flight control systems and numerical examples show the efficiency of our method.

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Key words: Inverse problem, flight control system, partial eigenstructure assignment, incomplete eigenvector.

1. Introduction

Consider the first-order linear control system

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1.1}$$

where $A \in \mathbb{R}^{n\times n}$ is the state matrix, $B \in \mathbb{R}^{n\times p}$ the control matrix, and $u(t) \in \mathbb{R}^p$ the control vector. The state feedback control is a vibration control method. Choosing the control vector $u(t)$ as

$$u(t) = -F^\top x(t) \tag{1.2}$$

with an $F \in \mathbb{R}^{n\times p}$, we substitute (1.2) into (1.1) and obtain the closed-loop system

$$\dot{x}(t) = Ax(t) - BF^\top x(t).$$

Separating variables

$$x(t) = xe^{\lambda t}, \quad \lambda \in \mathbb{C}$$

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yields

$$L_c(\lambda)x = 0,$$

where $L_c(\lambda) = \lambda I - (A - BF^\top)$.

In recent years, a substantial attention have been paid to the improvement of dynamic properties of the aircraft flight control systems. The corresponding system models can be reduced to the eigenstructure assignment problem of a linear system [1,8,11,16,18–20]. It is well known that the dynamic behaviour of such systems and the time response depend on their eigenvalues and eigenvectors [10]. Therefore, the eigenstructure assignment problem is an important area in various research projects. In particular, developing multi-input control systems, Moore [16] proposed a method of altering closed-loop eigenvectors, which help to improve the transient response characteristics of control systems without changing their eigenvalues. Porter [18] introduced the eigenstructure assignment algorithm, which can make the closed-loop system to have given eigenvalues such that the eigenvectors can be assigned by linear elementary transformations. However, this algorithm is not applicable in the case of multiple or complex-conjugate eigenvalues. On the other hand, Geng [8] developed an eigenstructure algorithm for solving the assignment problem with multiple and complex-conjugate eigenvalues. Andry *et al.* [1] introduced a new numerical method for solving the eigenstructure assignment problem for linear systems. Sobel *et al.* [19,20] provided a method of the eigenstructure assignment problem for output feedback control system. For vibrating control systems, this problem is discussed in [3,13,22].

In practical engineering problems, only certain eigenpairs of open-loop systems can be unsuitable. Therefore, one can change only these eigenpair and keep the remaining ones unchanged [5, 11]. Such an approach is called the partial eigenstructure problem. Kim *et al.* [11] adopted the method of eigenvector subspace to solve the partial eigenstructure assignment problem and proposed the quadratic index to evaluate the system properties. Mu [17] presented an eigenstructure assignment approach combined with the design of flight control system. Nevertheless, all those methods require the complete assignment or the completion of the unmeasured eigenvector elements — i.e. one has to determine inverses of various matrices, which greatly increases the computational costs.

Taking into account a limited number of transducers used to measure the response of the structure, we note that usually the number of co-ordinates of the measured eigenvectors is substantially smaller than the number of the degrees of freedom of finite element models [15]. Therefore, it is necessary to extend the measured mode shapes to the same size as their analytical counterparts. This leads to a partial eigenstructure assignment problem with incomplete measured modal data, which is widely used in practical engineering calculation related to dynamic properties of aircraft flight control systems and to the work of anti-seismic systems [9].

PEAP Problem. *Given an $n \times n$ real diagonalizable matrix A and p self-conjugate eigenpairs $\{(\lambda_i, v_i)\}_{i=1}^p$, $p < n$ of the open-loop and incomplete measured eigenpairs $\{(\mu_i, y_i^{(m)})\}_{i=1}^p$, find real matrices $B, F \in \mathbb{R}^{n\times p}$ such that the closed-loop pencil*

$$L_c(\lambda) = \lambda I - \left(A - BF^\top\right)$$