An Adaptive Modal Discontinuous Galerkin Finite Element Parallel Method Using Unsplit Multi-Axial Perfectly Matched Layer for Seismic Wave Modeling

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Abstract. The discontinuous Galerkin finite element method (DG-FEM) is a high-precision numerical simulation method widely used in various disciplines. In this paper, we derive the auxiliary ordinary differential equation complex frequency-shifted multi-axial perfectly matched layer (AODE CFS-MPML) in an unsplit format and combine it with any high-order adaptive DG-FEM based on an unstructured mesh to simulate seismic wave propagation. To improve the computational efficiency, we implement Message Passing Interface (MPI) parallelization for the simulation. Comparisons of the numerical simulation results with the analytical solutions verify the accuracy and effectiveness of our method. The results of numerical experiments also confirm the stability and effectiveness of the AODE CFS-MPML.

1 Introduction

In the field of geophysics, numerical simulations are an important tool for understanding the physical phenomena occurring on and inside the Earth. Accurate and efficient numerical simulation methods utilizing observational data provide objective evaluations of

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important measurements, the results of which play a vital role in studying the state and physical properties of the Earth's interior. In particular, over the past few decades, the numerical simulation of seismic waves has achieved considerable success and has solved many physical problems.

The finite difference method (FDM) adopts a difference scheme to approximate derivatives and solve differential equations. Historically, it was the first numerical method widely used in seismological research [1–6], and it is easy to program and boasts high computational efficiency. However, in cases involving complex topography and diverse geological conditions, the adaptability of the grid often degrades its calculation accuracy and amplifies the computational cost.

The pseudo spectral method (PSM) follows finite differences and is used extensively in several seismological research problems [7,8]. The PSM applies the fast Fourier transform (FFT) to process spatial derivatives, transforms the derivative operation in the space domain into a multiplication operation in the wavenumber domain, and then exploits the time recursion format to solve ordinary differential equations (ODE) in the time domain. As a result of this process, the PSM greatly reduces the amount of necessary storage space compared with the FDM. However the PSM has difficulty dealing with nonzero and non-periodic boundary conditions, and parallel computing is not easy to implement.

The finite element method (FEM) is another important numerical simulation method for seismic waves, that equivalently converts differential problems into variational problems, and utilizes a limited number of non-overlapping elements to partition the solution area [9–12]. Nevertheless, although the FEM easily addresses the problems associated with complex boundaries and changes in physical parameters, it relies on the global formation and solution of a large-scale linear algebraic system of equations. Thus, this approach must calculate and store large amounts of data. Furthermore, the FEM exhibits poor parallelism.

Considering the above techniques and their drawbacks, the spectral element method (SEM), which combines the advantages of the PSM and FEM, has become one of the most widely used numerical methods for solving the problem of seismic wave propagation [13–16]. The basis function of the SEM inside an element takes the Lagrange function at a specific point (Gauss–Lobatto–Legendre point), and then selects the Gauss–Lobatto–Legendre integration criterion to turn the mass matrix into a diagonal matrix, thereby avoiding the need to find the inverse of a large matrix. Several free software implementations of the SEM, such as SPECFEM, SES3D and RegSEM [17–19], exist for this purpose.

Another technique is the finite volume method (FVM), which avoids solving the spatial derivatives of the field by replacing them with flux terms [20–22]. A major advantage of the FVM is that (in principle) it allows calculation elements to take on arbitrary shapes. Despite this flexibility, however,the FVM has not been used for large-scale seismic simulation problems to date.

Regarded as a combination of the FVM and SEM that can solve partial differential equations (PDEs) with high accuracy, the discontinuous Galerkin FEM (DG-FEM) was