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A Priori Subcell Limiting Approach for the FR/CPR Method on Unstructured Meshes

Guo-Quan Shi[1], Huajun Zhu[1,2] and Zhen-Guo Yan[1,*]

[1] *State Key Laboratory of Aerodynamics, Mianyang, Sichuan 621000, China.*
[2] *College of Aerospace Science and Engineering, National University of Defense Technology, Changsha, Hunan 410073, China.*

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Abstract. A priori subcell limiting approach is developed for high-order flux reconstruction/correction procedure via reconstruction (FR/CPR) methods on two-dimensional unstructured quadrilateral meshes. Firstly, a modified indicator based on modal energy coefficients is proposed to detect troubled cells, where discontinuities exist. Then, troubled cells are decomposed into nonuniform subcells and each subcell has one solution point. A second-order finite difference shock-capturing scheme based on nonuniform nonlinear weighted (NNW) interpolation is constructed to perform the calculation on troubled cells while smooth cells are calculated by the CPR method. Numerical investigations show that the proposed subcell limiting strategy on unstructured quadrilateral meshes is robust in shock-capturing.

AMS subject classifications: 65D05, 65M06, 65M60

Key words: Flux reconstruction/correction procedure via reconstruction, shock capturing, subcell limiting, unstructured meshes.

1 Introduction

Although there are many industrial flow solvers based on second-order numerical methods, they suffer from large dissipation and dispersion errors so that they are difficult to provide more accurate results for some complex flows [1]. Recently, high-order and high-resolution schemes have attracted increasing attention. Especially high order schemes on unstructured grids draw a lot of attention when the simulations involve complicated geometries, such as the k-exact finite volume (FV) methods, discontinuous Galerkin (DG)

*Corresponding author. *Email addresses:* `yanzhg@mail.ustc.edu.cn`, `zgyan@skla.cardc.cn` (Z.-G. Yan), `gqsss@mail.ustc.edu.cn` (G.-Q. Shi), `hjzhu@skla.cardc.cn` (H. Zhu)

methods, spectral volume/spectral difference (SV/SD) methods [1], CPR method [2, 3], and some others [4, 5].

Among these high order methods, the CPR method is a differential method, which is applicable to both unstructured meshes and structured meshes. It can recover a specific kind of DG, SD, as well as SV with appropriate choices of correction terms [2]. However, compared with DG and SV methods, the CPR method is more efficient, as some of expensive integration procedures are avoided [2, 6]. The CPR method is first proposed by Huynh in 2007 to solve hyperbolic conservation laws on the structured meshes, which was called the flux reconstruction (FR) method [2]. Wang and Gao extend FR to 2D triangular and hybrid meshes [3], proposing the lifting collocation penalty (LCP) method. Due to the close relationship between FR and LCP methods, the involved scholars named them the CPR method conformably [7]. The mathematical foundation [8–13] and simulations [14–16] using the CPR method have been widely studied.

The CPR method generates spurious numerical oscillations near the discontinuities. Therefore, some shock-capturing strategies need to be used to suppress numerical oscillations. One approach is adding an artificial viscosity term, which was first proposed by von Neumann and Richtmyer [17]. In 2006, Persson and Peraire applied it for shock-capturing in DG methods [18] and later Yu and Wang extended it to the FR method [19]. Recently, Yu and Hesthaven studied several artificial viscosity models within the DG framework [20]. The artificial viscosity method has good accuracy, but it increases the complexity of the equations. In addition, it is difficult to find universal parameters for different types of flows.

There also exist some limiters to regulate the numerical oscillations. The multi-dimensional limiting process (MLP) originally in the FV framework has superior characteristics in terms of accuracy, robustness, and efficiency [21]. Park and Kim extend MLP to the CPR method on unstructured meshes [22]. The MLP method can capture detailed flow structures in both continuous and discontinuous flow regions. However, the limited CPR loses its compactness, because the stencil involves all the cells around the vertices. The weighted essentially nonoscillatory (WENO) limiter of DG [23] also has a satisfactory shock-capturing performance, which gains many attractions [24–28]. Shu et al. also employed a WENO limiter on the CPR method, which can maintain high-order accuracy in smooth regions and control spurious numerical oscillations near discontinuities [29, 30].

Another approach is the hybrid method, which adopts high-order methods in smooth regions to maintain compactness and high resolution, while using FD, FV, or lower-order FE schemes in troubled regions to provide robust shock-capturing abilities. Cheng et al. presented multi-domain hybrid RKDG/WENO-FD schemes based on domain decomposition [31]. In general, these FD, FV or FE schemes used for shock-capturing reduce the polynomial degree, so the resolution has to be preserved by h-refinement [32–36]. Dumbser et al. proposed a DG/FV hybrid scheme, which divides a troubled cell into $2N+1$ uniform subcells (N is the polynomial degree of DG) [32, 33]. Guo et al. proposed a hybrid weighted compact nonlinear scheme and CPR (WCNS-CPR) scheme for simu-