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CLIPPING OVER DISSIPATION IN TURBULENCE MODELS

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We dedicate this work to Max Gunzburger.
He started us on this adventure and inspired us along the way.

Abstract. *Clipping* refers to adding 1 line of code $A \Leftarrow \min\{A, B\}$ to force the variable A to stay below a present bound B. Phenomenological clipping also occurs in turbulence models to correct for over dissipation caused by the action of eddy viscosity terms in regions of small scales. Herein we analyze eddy viscosity model energy dissipation rates with 2 phenomenological clipping strategies. Since the true Reynolds stresses are $O(d^2)$ (d = wall normal distance) in the near wall region, the first is to force this near wall behavior in the eddy viscosity by $\nu_{turb} \Leftarrow \min\{\nu_{turb}, \frac{\kappa}{T_{ref}} d^2\}$ for some preset κ and time scale T_{ref}. The second is Escudier's early proposal to clip the turbulence length scale in a common specification of ν_{turb}, reducing too large values in the interior of the flow. Analyzing respectively shear flow turbulence and turbulence in a box (i.e., periodic boundary conditions), we show that both clipping strategies do prevent aggregate over dissipation of model solutions.

Key words. Energy dissipation rate, turbulence.

1. Introduction

Clipping in scientific programming refers to adding 1 line of code to force a preset upper or lower bound such as $A \Leftarrow \min\{A, B\}$. As an example, the standard parameterization of an eddy viscosity coefficient is $\nu_{turb} = \mu l \sqrt{k}$ where μ is a constant, $l = l(x, t)$ is the model's turbulent length scale and $k = k(x, t)$ is the model's approximation to the turbulent kinetic energy. The $\sqrt{k}$ term in ν_{turb} is often implemented as $\sqrt{\max\{k, 0\}}$ clipping small negative k values. Phenomenologically deduced clipping occurs in turbulence models to correct for over dissipation caused by the action of eddy viscosity terms in regions of small velocity scales and is tested in numerical experiments. Herein we develop analytical support, analyzing model dissipation, for clipping in URANS (Unsteady Reynolds Averaged Navier Stokes) turbulence models, complementing phenomenology and numerical tests. The true Reynolds stresses are $\mathcal{O}(d^2)$ ($d = \inf\{|x - y| : y \in \partial\Omega\}$, the wall normal distance) in the near wall region. The first clipping strategy we analyze is to force this $\mathcal{O}(d^2)$ behavior in the eddy viscosity by $\nu_{turb} \Leftarrow \min\{\nu_{turb}, \frac{\kappa}{T_{ref}} d^2\}$ for some preset and non-dimensional κ and time scale T_{ref}. The second clipping strategy acts on the model's turbulence length scale, the variable l in $\nu_{turb} = \mu l \sqrt{k}$. We analyze Escudier's clipping of this turbulence length scale in the interior. Analyzing in the first and second cases respectively shear flow turbulence and turbulence in a box (i.e., periodic boundary conditions), we show that *both clipping strategies prevent aggregate over dissipation of eddy viscosity model solutions.*

A wide variety of eddy viscosity models exist. Current practice, summarized in Wilcox [40], favors eddy viscosity based, URANS models arising from time averaging, e.g., Durbin and Pettersson Reif [12] (p. 195). Following, for example Mohammadi and Pironneau [26] and Wilcox [40] (p.37 Eq 3.9), the model velocity

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$v(x,y,z,t) \simeq \overline{u}(x,y,z,t)$ approximates the finite time average[1] $\overline{u}$ of the Navier-Stokes velocity u

$$(1) \qquad \overline{u}(x,y,z,t) = \frac{1}{\tau}\int_{t-\tau}^{t} u(x,y,z,t')dt' \text{ and fluctuation } u' := u - \overline{u}.$$

Causality requires the time window, $t-\tau < t' < t$, to stretch backwards as above so present velocities do not depend on future forces. The associated turbulent kinetic energy is then $\frac{1}{2}\overline{|u-\overline{u}|^2}$. Averaging the Navier Stokes equations (NSE) yields the system $\nabla \cdot \overline{u} = 0$ and

$$\overline{u}_t + \overline{u}\cdot\nabla\overline{u} - \nabla\cdot(2\nu\nabla^s\overline{u}) - \nabla\cdot R(u,u) + \nabla p = \frac{1}{\tau}\int_{t-\tau}^{t} f(x,y,z,t')dt',$$
$$\text{where} \quad R(u,u) = \overline{u}\otimes\overline{u} - \overline{u\otimes u}.$$

Here ν is the kinematic viscosity, p is a pressure, f is the body force, $\nabla^s u$ is the symmetric part of ∇u, U is a global velocity scale, L is a global length scale and the Reynolds number is $\mathcal{R}e = LU/\nu$. This equation is not closed. Models replace $R(u,u)$ by terms that only depend on $\overline{u}$. For time window τ sufficiently large (and $t > \tau$) time dependence disappears from the equation and steady state RANS models result. For time window small, τ can be treated as a small parameter in $R(u,u)$ and models can be derived by asymptotics. Herein we consider URANS modelling for intermediate τ.

The main URANS model used in practical turbulent flow predictions is of eddy viscosity type. Its velocity $v(x,y,z,t) \simeq \overline{u}(x,y,z,t)$ satisfies

$$(2) \quad v_t + v\cdot\nabla v - \nabla\cdot(2[\nu+\nu_{turb}]\nabla^s v) + \nabla p = \frac{1}{\tau}\int_{t-\tau}^{t} f(x,y,z,t')dt',\ \nabla\cdot v = 0.$$

Herein we first analyze in Section 2 the near wall behavior of the general eddy viscosity model, i.e., any choice of $\nu_{turb}(x,y,z,t) \geq 0$. The eddy or turbulent viscosity $\nu_{turb}(\geq 0)$ must be specified. Section 3 analyzes the away from wall behavior of the common, $1-$equation specification $\nu_{turb} = \mu l\sqrt{k}$ where $k(x,y,z,t)$ satisfies the classical equation for the turbulent kinetic energy.

A classical turbulent viscosity specification is the Smagorinsky-Ladyzhenskaya $0-$equation model $\nu_{turb} = (0.1\delta)^2|\nabla^s v|$ where $\delta =$ selected length scale, analyzed by Du, Gunzburger and Turner in [10], [36]. The classic $1-$equation model of Prandtl and Kolmogorov is analyzed in Section 3. $2-$equation models add a second, phenomenologically derived equation that determines the $1-$equation turbulence length scale l. In all these cases, the total *model energy dissipation rate per unit volume* is

$$(3) \qquad \varepsilon_{\text{model}}(v) := \frac{1}{|\Omega|}\int_\Omega 2[\nu+\nu_{turb}]|\nabla^s v(x,y,z,t)|^2 dx.$$

A common failure mode of eddy viscosity models is over dissipation, either producing a lower $\mathcal{R}e$ flow or even driving the solution to a nonphysical steady state. This occurs due to the action of the turbulent viscosity term near walls or on interior small scales. We study over dissipation here through interrogation of the above model energy dissipation rate. A wide range of boundary conditions occur in practical flow simulations. Herein we focus on two: shear boundary conditions to study turbulence generated by near wall flows (Section 2) and $L-$periodic to study turbulence dynamics away from walls (Section 3).

[1]The time average can occur after ensemble averaging plus an ergodic hypothesis. URANS models are also constructed ad hoc simply by adding $\frac{\partial v}{\partial t}$ to a RANS model.