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2D Random Approximation Method*

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Abstract H. Robbins and S. Monro studied the stochastic approximations of one-dimensional system. In this paper, we present the stochastic approximation method of 2D system.

Keywords 2D systems, Stochastic approximation, Mathematical expectation, Convergence.

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1. Introduction

H. Robbins and S. Monro studied the stochastic approximation of one-dimensional system in [24]. However, there are a large number of 2D stochastic systems in stochastic fluid mechanics [2], especially in the diffusion of random electronic gas in magnetic region [29], random information flow [18, 28] and other engineering fields. Recently, scholars have shown interest in the method of stochastic approximation, a lot of work has been done [3, 7, 8, 14, 30] and they are of great significance. The stochastic approximation method can be used to solve some random or random problems as well as some deterministic mathematical problems [16, 23, 31].

To a less extent, we investigate methods like stochastic average gradient [25], which performs well on objectives that are strongly-convex, and stochastic variance reduced gradient [11]. Fort et al. [6] establish results on the geometric ergodicity of hybrid samplers and in particular for the random-scan Gibbs sampler. Zhao and Wang [32], Jansen et al. [10] and Kriegesmann [13] estimate the statistical moments of the compliance by Monte Carlo approximations. Sparse polynomial chaos expansions [1, 4, 5, 9] can be used to reduce the computational cost, but the computational cost associated with this approach becomes prohibitive for a large number of problems with uncertain inputs. [21, 22, 26, 27] use the stochastic approximation algorithm of Polyak-Ruppert averaging to favor the performance of stochastic approximation. Convergence results for mini-batch EM and SAEM algorithms appear recently in [17, 19] and [12] respectively.

Moreover, the standard method for stochastic root-finding problems is stochastic approximation [15, 20, 24]. In these 2D stochastic systems, take α as a constant, consider a region D on the plane R^2, and find the equation satisfied by the unknown

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measurable regression function $M(x)$ with error:

$$M(x) = \alpha. \tag{1.1}$$

The null point $x = \theta$ of the above equation is an ubiquitous and important problem in system identification, adaptive control, pattern recognition, adaptive filtering and neural network and other fields.

Generally, $M(x)$ represents a mathematical expected value at time x of a certain experiment, which is an unknown function. However, for any x, the value of $M(x)$ is measurable, and assume that $M(x)$ is a monotone function of x in an unknown experiment. To obtain the null point $x = \theta$ of (1.1), we need to design an algorithm to determine a series of values $\{x_{mn}\}_{m,n\geq 0}$ in the region D of the plane R^2, which are

$$\begin{aligned} &x_{00}, x_{01}, , x_{02} \cdots\cdots, \\ &x_{10}, x_{11}, x_{12} \cdots\cdots, \\ &x_{20}, x_{21}, x_{22} \cdots\cdots, \\ &\cdots\cdots\cdots\cdots\cdots\cdots, \end{aligned}$$

these values satisfy in a probabilistic way, $\lim_{\substack{m\to\infty \\ n\to\infty}} x_{mn} = \theta$.

As mentioned above, for any x_{mn}, $M(x_{mn})$ can be measured, so it can provide information for the next measured value. Notice that there are two coordinate positions for the next measured value related to x_{mn}, $(m+1, n)$ and $(m, n+1)$, so there are two points,

$$x_{m+1,n}, x_{m,n+1}. \tag{1.2}$$

However, for the next measured value, considering the convenience of researching problem, we usually regard x_{mn} as the value to be measured in the next step. Therefore, there are two values related to x_{mn} directly, which are

$$x_{m-1,n}, x_{m,n-1}. \tag{1.3}$$

Define $D = \left\{ (m,n)/ \begin{matrix} m \geq 0 \\ n \geq 0 \end{matrix} \right\}$, then $D \subset R^2$. Besides, for any $(m, n) \in D$, considering a mathematical sequence $\{x_{mn}\}_{m,n\geq 0}$, and the boundary values x_{m0}, x_{0n} are known. Therefore, $M(x_{m0})$ and $M(x_{0n})$ are determined. Thus, for any value x_{mn} in D, the points directly connected with x_{mn} have two coordinate positions, $(m-1, n)$ and $(m, n-1)$.