

Positive Periodic Solutions for a Single-species Model with Delay Weak Kernel and Cycle Mortality*

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Abstract In this paper, by using the Krasnoselskii's fixed-point theorem, we study the existence of positive periodic solutions of the following single-species model with delay weak kernel and cycle mortality:

$$x'(t) = rx(t) \left[1 - \frac{1}{K} \int_{-\infty}^t \alpha e^{-\alpha(t-s)} x(s) ds \right] - a(t)x(t),$$

and get the necessary conditions for the existence of positive periodic solutions. Finally, an example and numerical simulation are used to illustrate the validity of our results.

Keywords Positive periodic solutions, Single-species model, Delay, Cycle mortality.

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1. Introduction

As we all know, the application of delay differential equations in population dynamics can be traced back to the 1920s, and in the past 100 years, the theory of population dynamics has achieved significant development. For example, see [2, 3, 9, 10]. In 1980, Gurney et al. [8] studied the delayed Nicholson's blowflies equation

$$N'(t) = PN(t - \tau)e^{-\alpha N(t-\tau)} - \delta N(t), \quad (1.1)$$

where $N(t)$ represents the population of mature adults at time t , $\frac{1}{\alpha}$ denotes the population size at which the complete population reproduces at its maximum rate, P denotes the maximum possible per capita egg production rate, $\tau > 0$ is a delay term and $\delta > 0$ is the mortality rate.

Consider the different practical conditions, model (1.1) is generalized to more general models. In 2008, Li et al. [11] used the Krasnoselskii's fixed-point theorem to prove the existence of the positive periodic solution of the following generalized

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Nicholson's blowflies model:

$$x'(t) = -\delta(t)x(t) + \sum_{i=1}^m p_i(t)x(t - \tau_i(t))e^{-q_i(t)x(t - \tau_i(t))}, \quad t \geq 0.$$

In [1], by using the Schauder's fixed-point theorem, they study the existence of positive periodic solutions of Nicholson's blowflies differential equation with iterative harvest function:

$$N'(t) = p(t)N(t - \tau)e^{-\gamma(t)N(t - \tau)} - a(t)N(t) - qN(t - \tau)E(t, N(t), \dots, N^{[n]}(t)),$$

where E denotes the harvesting effort, defined as the intensity of the human activities to harvest the flies and $q \geq 0$ is the so-called the catchability coefficient, which express the fraction of the population that is removed by one unit of harvesting effort.

In 1934, Volterra [14] proposed a more accurate model based on the Logistic model:

$$\frac{dN(t)}{dt} = N(t) \left[1 - \frac{1}{K} \int_{-\infty}^t G(t-s)N(s)ds \right],$$

where $G(t)$ called the delay kernel, is a weighting factor which says how much emphasis should be given to the size of the population at earlier times to determine the present effect on resource availability. The delay kernel is usually normalized so that $\int_0^{\infty} G(u)du = 1$. Two special cases including

$$G(u) = \alpha e^{-\alpha u},$$

$$G(u) = \alpha^2 u e^{-\alpha u}$$

are called weak delay kernel and strong delay kernel.

Based on the discussions above, in this paper, we consider a single-species model with delay weak kernel and death term as follows:

$$x'(t) = rx(t) \left[1 - \frac{1}{K} \int_{-\infty}^t \alpha e^{-\alpha(t-s)} x(s) ds \right] - a(t)x(t), \quad (1.2)$$

where $r > 0$ is the intrinsic rate of growth for population, $x(t)$ is the size of the population at time t , $K > 0$ is the population's carrying capacity and $a(t)$ is the death rate and we assume that (H) holds in this article:

(H) $a : \mathbb{R} \rightarrow (0, 1)$ is T -periodic continuous coefficients satisfying $a(t) = a(t+T)$ for $t \in \mathbb{R}$.

In recent years, the existence of positive T -periodic solutions of periodic ecological models with delays have been studied by many authors. For example, see [4–6, 12, 13, 15, 16] and references therein. However, it is worth mentioning that there is no conclusion about the existence of positive T -periodic solutions of equation (1.2). Therefore, our results are completely new.

The paper is organized as follows: In Section 2, we give a simple analysis of equation (1.2) and some definitions and lemmas are given. In Section 3, we use the Krasnoselskii's fixed-point theorem to obtain the positive periodic solution of the model (1.2) under some given conditions. In Section 4, an example is given to illustrate our results obtained in the previous section.