

A Reflection on Teaching Mathematical Proofs to First-Year Undergraduates

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Abstract

Writing a mathematical proof is an essential skill for qualified science, technology, engineering, and mathematics (STEM) students, especially those majoring in mathematics, data science, artificial intelligence (AI), and computer science. However, many first-year students complain that it is tremendously difficult for them to learn mathematics well, particularly mathematics proofs. New teaching staff are also often frustrated during their first couple of years of teaching undergraduates mathematics, wondering why something so seemingly simple is so difficult to teach. Although most universities have experienced this problem, there has been little deep reflection and investigation on why learning of mathematical proofs is so difficult for first-year undergraduates and how to mitigate such difficulties. The aim of this paper is to fill such a gap by reflections and investigation which focus on the recognition and learning of the mathematical proof process.

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Importance of Mathematical Proofs

Role of Proofs

A proof is a valid argument that establishes the truth of a (mathematical) statement. An argument is a compound statement with some premises (assumptions or facts) and a conclusion. According to this definition, a proof is a way of generating new knowledge based on existing knowledge or assumptions. Almost all new mathematical results are established by a rigorous mathematical proof. A mathematical proof is not only important to establish significant results as theorems, but also for many other applications. It can be used to design a debugging system for computer programs, to make inferences in artificial intelligence (AI), to show whether a system specification is consistent, and so on. It is well accepted that a mathematical proof is one of the foundations of computer science and AI.

Proving is seen as an essential skill for science, technology, engineering, and mathematics (STEM) students to acquire. Bell (1976) argues that a mathematical proof can be used for verification, illumination, and systematization. It can also be used for discovery or communication or simply as an intellectual challenge (De Villiers, 1990, 2003). Balacheff (2010) points out that a mathematical proof can also be

used for the construction of empirical theory, exploration of the meaning of a definition or the consequences of an assumption, and the incorporation of a well-known fact into a new framework, thus enabling it to be viewed from a fresh perspective.

Teaching Proofs Is Important

Teaching proofs is not only important for students majoring in STEM, but for almost any subject, as has been discussed by many researchers. We believe that teaching mathematics properly is critically important for the following reasons.

First, writing a proof helps students to understand basic definitions, concepts, and theory. A proof can help students learn about why a theory holds from some basic or simple facts. Once students understand the reason and mechanism underlying a proof, they can remember results more deeply and for longer, as the learning of mathematics usually requires students to both understand and memorize. As memorization with proper understanding can accelerate students' learning (Zhang et al., 2019), a proof can help students to deepen their understanding of underlying mathematical ideas and reinforce their memorization (Lesseig et al., 2019).

Second, a proof can help students to build up a knowledge graph (KG) which is easily taken away. A KG is like a bunch