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Multipliers of Dirichlet-Type Subspaces of Weighted Bloch Spaces

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Abstract. In this paper, we characterize the multipliers on the intersection of the Dirichlet-type space and the logarithmic Bloch space, and the intersection of the Dirichlet-type space and the logarithmic space of analytic functions of bounded mean oscillation.

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1 Introduction

Let $\mathbb{D}$ denote the unit disk of the complex plane $\mathbb{C}$ and let $\partial\mathbb{D}$ be the boundary of $\mathbb{D}$, the unit circle. Denote by $\mathcal{H}(\mathbb{D})$ the space of all analytic functions in $\mathbb{D}$. Let $f\in\mathcal{H}(\mathbb{D})$, for $0<p<\infty$, $0<r<1$, set

$$M_p^p(r,f)=\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^p d\theta,$$

and

$$M_\infty(r,f)=\sup_{|z|=r}|f(z)|.$$

The Hardy space $\mathcal{H}^p$ $(0<p\leq\infty)$ is defined by those $f\in\mathcal{H}(\mathbb{D})$ such that

$$\|f\|_{H^p}=\sup_{0<r<1}M_p(r,f)<\infty.$$

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For the detail of the theory about the Hardy space $\mathcal{H}^p$, we refer the reader to [8, 11, 22].

The Dirichlet-type space $\mathcal{D}^p_\alpha$ is the set of all $f\in\mathcal{H}(\mathbb{D})$ such that

$$\|f\|^p_{\mathcal{D}^p_\alpha}=\int_{\mathbb{D}}|f'(z)|^p dA_\alpha(z)<\infty,$$

where $\alpha>-1$, $dA_\alpha(z)=(\alpha+1)(1-|z|^2)^\alpha dA(z)$, and $dA(z)=\frac{1}{\pi}dxdy$ is the normalized Lebesgue area measure. It is well known that if $p<\alpha+1$, then $\mathcal{D}^p_\alpha=A^p_{\alpha-p}$, the Bergman space (see [9]). If $p>\alpha+2$, then $\mathcal{D}^p_\alpha\subseteq\mathcal{H}^\infty$. Therefore, when $\alpha+1\le p\le\alpha+2$, $\mathcal{D}^p_\alpha$ is a proper Dirichlet-type space. In this paper, we are interested in the space $\mathcal{D}^p_{p-2+s}$ when $0<s<1$.

The Bloch space $\mathcal{B}$ consists of those $f\in\mathcal{H}(\mathbb{D})$ for which

$$\|f\|_{\mathcal{B}}=|f(0)|+\sup_{z\in\mathbb{D}}(1-|z|^2)|f'(z)|<\infty.$$

The logarithmic Bloch space $\mathcal{B}_{\log}$ is defined by those $f\in\mathcal{H}(\mathbb{D})$ for which

$$\|f\|_{\mathcal{B}_{\log}}=|f(0)|+\sup_{z\in\mathbb{D}}(1-|z|^2)|f'(z)|\log\frac{2}{1-|z|^2}<\infty.$$

The space of analytic functions of bounded mean oscillation $BMOA$ (see [12]) is the set of all functions $f\in\mathcal{H}^1$ such that

$$\|f\|^2_{BMOA}=\sup_{a\in\mathbb{D}}\int_{\mathbb{D}}|f'(z)|^2(1-|\varphi_a(z)|^2)dA(z)<\infty,$$

where $\varphi_a(z)=\frac{a-z}{1-\bar{a}z}$ is the Möbius transformation which interchanges the point a and 0.

The logarithmic space $BMOA_{\log}$ consists of those $f\in\mathcal{H}^1$ for which

$$\|f\|^2_{BMOA_{\log}}=\sup_{a\in\mathbb{D}}\left(\log\frac{2}{1-|a|}\right)^2\int_{\mathbb{D}}|f'(z)|^2(1-|\varphi_a(z)|^2)dA(z)<\infty.$$

If $f\in BMOA_{\log}$, then the growth estimate for f is given by (see Lemma 2.4 in [19])

$$|f(z)|\le C\log\log\frac{4}{1-|z|}\|f\|_{BMOA_{\log}},\qquad z\in\mathbb{D}.$$

It is known that the spaces $BMOA$, $\mathcal{B}_{\log}$ and $BMOA_{\log}$ are subspaces of the Bloch space $\mathcal{B}$.

Given $g\in\mathcal{H}(\mathbb{D})$, the multiplication operator M_g is defined by

$$M_g f(z)=g(z)f(z),\qquad z\in\mathbb{D},\quad f\in\mathcal{H}(\mathbb{D}).$$

Let X,Y be Banach spaces of analytic functions in $\mathbb{D}$. Let $M(X,Y)$ be the space of multipliers from X to Y, in other words,

$$M(X,Y)=\{g\in\mathcal{H}(\mathbb{D}): fg\in Y,\ \forall f\in X\}.$$