

The Value of a Two-Sided Real Swaption

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Abstract. In this paper we establish a mathematical model for two-sided matching swaption problems in real world. We find a necessary matching condition of the two-sided swaption and deduce analytical formulations of the swaptions in virtue of pricing kernel methods. Also we explore the optimal exercise boundary and properties of the swaptions for investment decision making.

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Key words: Real option, swaption, two-sided matching, optimal exercise boundary.

1 Introduction

Options in finance are contracts which give the holders rights but not obligations to buy or sell certain quantity of assets with certain strike prices at given time [8]. The option holders must compensate the underwriters certain premiums, namely option prices, at the beginning in order to obtain the rights. Call options give their holders buying rights, while put options give their holders selling rights. European options are those only can be exercised on the maturity dates, while American options can be exercised at any time before expiration.

Swaption is an option that gives its holder the rights to enter a swap contract involving the exchange of pre-agreed cash flows of two financial instruments [12, 14, 15]. Swaptions are common financial tools in practice, like interest rate swaption, equity swaption and futures swaption etc, which have been studied a lot [15]. Take the interest rate swaption in finance for example, it comes in two main types: a payer swaption and a receiver swaption. In the payer swaption, the purchaser has the right but not obligation to enter

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into an interest swap that pays fixed interest rate and receive floating rate of a notional principal in future time nodes. While a receiver swaption is the opposite, i.e., the holder has the option to receive the fixed rate and pay the floating rate in the given time nodes. Since swaptions are two-sided over-the-counter (OTC) contracts which are not standardized, thus both the buyer and the seller need to negotiate the price of the swaption.

In this paper, we discuss a type of American real swaption, that is two representatives[†] sign a swaption contract with each other to swap their real assets[‡] in the future, wherein the assets are growing continuously and stochastically with endogenous consumption [11, 13]. Classical finance theory seldom refers to this problem, since the real market is incomplete without enough hedging tools and ordinary risk-neutral pricing methodology does not work. Besides, the swaption involves in two-sided matching and dynamic gaming which are quite profound in mathematics and economics. Nevertheless, swaptions are widespread in practice, and many real investment and decision making can be deemed swaption problems, like cross-shareholding, mate choice [3], and China-Iraq agreement etc. Therefore, the two-sided real swaption problems are very important and worthy of study. Hereafter, we only focus on the pricing of the real swaption, and assume the two-sided matching processes of the swaption are naturally finished. Such assumption does not affect the pricing essentially. For the two-sided matching, the Nobel Laureates Alvin E. Roth and his cooperator research a lot through game theory in [1].

The contribution of the paper is that we firstly establish a real swaption model in mathematics which applies to many problems in practice, and succeed in getting the value of the swaption with the aid of pricing kernel method. We obtain a necessary matching condition of the swaption, although we keep away from the matching and gaming processes in the swaption model. Besides, optimal exercise boundary and a lot of propositions of the options are deduced. All of these results are helpful for real investment and decision making in practice.

The remainder of the paper is organized as follows. Section 2 establishes a real swaption model in virtue of pricing kernel method. Section 3 deduces the pricing formulations and properties of the options in the swaption. All the proofs of the main results in Section 3 are shown in Section 4.

2 Modeling of real swaption

To establish a real swaption model from a quantitative angle, we assume a dynamic process for the real asset of a representative with endogenous consumption. The representative has the right to exchange its real asset with another one in future dates after signing a swaption contract, and vice versa.

Different from standard assets in financial market, we could not hedge and replicate the real asset since the market is incomplete, thus ordinary risk-neutral pricing methods

[†]The representative can be a person, a company, or an economy, etc.

[‡]The real asset can be a person's endowment or a company's equity, or an economy's resource.