CSIAM Trans. Appl. Math.
doi: 10.4208/csiam-am.SO-2021-0039

Vol. **3**, No. 4, pp. 626-661
December 2022

Relaxed Alternating Minimization Algorithm for Separable Convex Programming with Applications to Imaging

Shuangshuang Wu[1], Yuchao Tang[1,*] and Tieyong Zeng[2]

[1] *Department of Mathematics, Nanchang University, Nanchang 330031, China.*
[2] *Department of Mathematics, The Chinese University of Hong Kong, Shatin, Hong Kong, China.*

Received 14 August 2021; Accepted 8 April 2022

Abstract. We propose a relaxed alternating minimization algorithm for solving two-block separable convex minimization problems with linear equality constraints, where one block in the objective functions is strongly convex. We prove that the proposed algorithm converges to the optimal primal-dual solution of the original problem. Furthermore, the convergence rates of the proposed algorithm in both ergodic and nonergodic senses have also been studied. We apply the proposed algorithm to solve several composite convex minimization problems arising in image denoising and evaluate the numerical performance of the proposed algorithm on a novel image denoising model. Numerical results for both artificial and real noisy images demonstrate the efficiency and effectiveness of the proposed algorithm.

AMS subject classifications: 47H05, 90C25, 65K05

Key words: Alternating minimization algorithm, strongly convex function, total variation, image denoising.

1 Introduction

In this study, we focus on solving the following convex minimization problem:

$$\begin{aligned} \min_{x\in H_1, y\in H_2} & f(x)+g(y) \\ \text{s.t.} \quad & Ax+By=b, \end{aligned} \tag{1.1}$$

*Corresponding author. *Email addresses:* `1628941043@qq.com` (S. Wu), `yctang@ncu.edu.cn` (Y. Tang), `zeng@math.cuhk.edu.hk` (T. Zeng)

where H_1, H_2, and G are real Hilbert spaces, $f:H_1\to(-\infty,+\infty]$ and $g:H_2\to(-\infty,+\infty]$ are proper lower-semicontinuous (lsc) convex functions, $b\in G$, $A:H_1\to G$ and $B:H_2\to G$ are nonzero bounded linear operators. Throughout this study, we assume that f is σ-strongly convex for some $\sigma>0$. Furthermore, many problems from different areas are special cases of (1.1), such as the projection onto the intersection of convex sets [1] and the resource allocation problem [2]. In the following, we present several concrete examples of signal and image processing.

Example 1. (Total variation image denoising problem) The total variation image denoising problem was first proposed by Rudin, Osher, and Fatemi [3], which takes the form of

$$\min_x \frac{1}{2}\|x-u\|^2+\lambda\|x\|_{TV}, \tag{1.2}$$

where $u\in R^{m\times n}$ is the observed noisy image, $\|x\|_{TV}$ denotes total variation, and $\lambda>0$ is the regularization parameter. (1.2) is often referred to as the ROF model. The total variation $\|x\|_{TV}$ can be represented by the composition of a convex function φ and a first-order difference operator $L:R^{m\times n}\to R^{m\times m}\times R^{m\times n}$, that is, $\|x\|_{TV}=\varphi(Lx)$ [4,5]. Then, (1.2) is a special case of (1.1).

Example 2. (Constrained total variation image denoising problem) Considering the prior information of pixel values in digital images is often bounded by $0\le x\le 1$ or $0\le x\le 255$. Let C be a nonempty closed convex set. The following constrained ROF model was considered in [6–9]:

$$\begin{aligned} &\min_x \frac{1}{2}\|x-u\|^2+\lambda\|x\|_{TV}\\ &\text{s.t.}\ \ x\in C. \end{aligned} \tag{1.3}$$

Example 3. (Mixed Gaussian-impulse noise problem) In many practical applications, the noise distribution is relatively complex, and Gaussian noise cannot be accurately simulated. Many studies have focused on the removal of mixed Gaussian noise. The following mixed Gaussian-impulse noise problem, in which the total variation as a regularization function, was independently studied in [10–13]:

$$\min_x \frac{1}{2}\|x-u\|^2+\mu\|x-u\|_1+\lambda\|x\|_{TV}, \tag{1.4}$$

where $\mu>0$ and $\lambda>0$ are regularization parameters. (1.4) is often referred to as the L2-L1-TV model. In addition, box constraints or nonnegativity constraints can also be added to (1.4) to improve its performance.

Example 4. (Mixed Poisson-Gaussian noise problem) Due to the influence of photon counting and thermal noise, the observed image is usually destroyed by mixed Poisson-Gaussian noise. Many studies have focused on mixed Poisson-Gaussian denoising problems. See, for example [14–17]. In particular, Thanh and Dvoenko [18] considered the