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Modified Legendre Rational Spectral Method for Burgers Equation on the Whole Line

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Abstract. In this paper, we propose a spectral method for the Burgers equation using the modified Legendre rational functions, and prove its generalized stability and convergence. Numerical results demonstrate the efficiency of the new approach.

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Key words: Burgers equation, spectral method, modified Legendre rational functions, problem on the whole line.

1 Introduction

Burgers equation is a fundamental mathematical model in many application areas, such as fluid dynamics [1,2], nonlinear acoustics [3], gas dynamics [4,5], traffic flow [6,7]. The Burgers equation on the whole line is of the form

$$\begin{cases} \partial_t U(x,t)+\frac{1}{2}\partial_x(U^2(x,t))-\mu\partial_x^2 U(x,t)=f(x,t), & x\in\Lambda,\ 0<t\leq T,\\ \lim\limits_{x\to\pm\infty} U(x,t)=0, & 0\leq t\leq T,\\ U(x,0)=U_0(x), & x\in\Lambda, \end{cases} \tag{1.1}$$

where $\Lambda=\{x|-\infty<x<\infty\}$, $T>0$, $\mu>0$ is the kinetic viscosity, $f(x)$ is the source term, $U_0(x)$ is the initial state.

The problem (1.1) defined on unbounded domain. This brings a challenge to design effective numerical methods to overcome the unboundedness of the physical domain and make theoretical analysis. Some authors set artificial boundary conditions and applied finite difference methods [8–12], or finite element methods [10, 13–16], or spectral

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and spectral collocation methods [17–25] to solve the Burgers equation on the trancated domain. However, they induce additional errors. Spectral methods associated with the orthogonal polynomials/functions on unbounded domain are natural candidates for problem (1.1) [26,27]. Guo, Shen and Xu [28] proposed spectral and pseudospectral methods using Hermite functions for solving problems defined on the whole line. Xiang and Wang [29] developed a generalized Hermite spectral method to solve the problem on unbounded domains. Guo and Ma [30] designed a composite Legendre-Laguerre spectral method for solving problems on unbounded domain. Guo and Wang [31] proposed a composite generalized Laguerre-Legendre spectral method for problems in an infinite channel. Whereas, these composite methods bring inconvenient in theoretical analysis and computation.

Another way for solving problems on unbounded domains is using spectral methods associated with rational functions on the corresponding domains. Boyd [32] and Christov [33] designed some spectral methods for linear problems on infinite intervals by using certain mutually orthogonal systems of rational functions. Boyd [34] proposed a spectral method by rational Chebyshev functions on semi-infinite interval. Guo, Shen and Wang [35] constructed Chebyshev rational spectral and pseudospectral approximations and applied to problems on semi-infinite interval. Guo and Shen [36] introduced modified Legendre rational functions and related spectral and pseudospectral methods for solving partial differential equations on the half line. Guo and Wang [37] investigated Legendre rational approximation and used to a model problem on the whole line. Wang and Guo [38] developed modified Legendre rational spectral method for the problem on the whole line.

The purpose of this paper is to design a spectral method using the modified Legendre rational functions for the Burgers equation (1.1). The merits of our approach are as follows:

- The modified Legendre rational function approximates solution of the Burgers equation on the whole line naturaly.

- The uniform weight function $\omega(x)\equiv 1$ makes the numerical solution keeps the same conservation as the genuine solution and simplifies the actual computation and analysis.

- The numerical solutions possess spectral accuracy in space.

The paper is organized as follows. In the next section, we recall some approximation results of the modified Legendre rational functions. In Section 3, we propose a spectral scheme for the Burgers equation and analyze its stability and convergence. In Section 4, we present some numerical results to demonstrate the efficiency of the algorithm. The final section is for some concluding remarks.