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Averaging Principle for Stochastic Tidal Dynamics Equations

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Abstract. In this paper, we aim to establish a strong averaging principle for stochastic tidal dynamics equations. The averaging principle is an effective method for studying the qualitative analysis of nonlinear dynamical systems. Under suitable assumptions, utilizing Khasminkii's time discretization approach, we derive a strong averaging principle showing that the solution of stochastic tidal dynamics equations can be approximated by solutions of the system of averaged stochastic equations in the sense of convergence in mean square.

AMS subject classifications: 60H15, 35Q35, 60H30

Key words: Stochastic tidal dynamics equations, averaging principle, strong convergence.

1 Introduction

For hundreds of years, ocean tides have been a source of interest for many physicists and mathematicians. Historically, Newton first gave a mathematical explanation of ocean tides and Laplace established the hydrodynamic equations for ocean tides, we refer the readers to the literature [12] for a complete history and theoretical description of tides. Over the last few decades, this field

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has developed further with the help of computer and satellite technology and is used in a wide and diverse range of fields such as geophysics, atmospheric science and communications, just to mention a few. In this paper, we will consider the tidal dynamics model proposed by Marchuk and Kagan [15]. In the monograph [15], Marchuk and Kagan constructed the tidal dynamics model from the three-dimensional Navier-Stokes equations by integrating along the z-axis (the vertical direction) and then by considering the model on a rotating sphere, which is a generalisation of the Laplace model.

Due to modelling complexity, it is intrinsically difficult and challenge to study the tidal dynamics equations which are highly nonlinear partial differential equations of parabolic-hyperbolic type. Let us give a brief review of results in the literature. In Manna *et al.* [14], the authors obtained the existence and uniqueness of weak solutions of the deterministic tide dynamics equations and the existence and uniqueness of strong solutions of the stochastic tide dynamics equations with additive Gaussian white noise. The existence, uniqueness, large deviation principle and moderate deviation principle for stochastic tidal dynamics equations driven by multiplying Gaussian noise have been studied in [9, 19]. The authors in [1] established the existence of optimal controls for stochastic tidal dynamics equations driven by Lévy noise. For further studies regarding stochastic tidal dynamics equations, interested readers are referred to [16, 17, 23] and references therein.

On the other hand, averaging principle is an effective approach for studying dynamical systems involving highly oscillating components. Under certain assumptions, the highly oscillating components can be averaged out to generate an averaged dynamical system, which is comparably easier for analysis which governs the evolution of the original system over long time scales. The averaging principle for deterministic dynamics initially established by Krylov and Bogolyubov indeed provides a powerful and efficient tool for investigating the properties of highly complex and nonlinear dynamical systems. Averaging principle for stochastic differential equations was first derived by Khasminskii in [11]. The fundamental idea of the stochastic averaging principle is to derive averaged stochastic differential equations and establish approximation of the averaging solutions to the solutions of the original equations, so that the original complex stochastic differential equations could be analysed via the corresponding easier averaged equations. To date, there are extensive literatures concerning stochastic averaging principle for finitely dimensional and infinitely dimensional stochastic systems, see, e.g., [2–4, 7, 8, 10, 18, 20–22] and the references therein. Motivated by all the above mentioned works, in this paper, we want to establish a strong averaging principle for the stochastic tidal dynamics equations in which we derive