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Dynamic Analysis of an Impulsive Chemostat Model with Microbial Competition and Nonlinear Perturbation*

Yue Dong[1] and Xinzhu Meng[1,†]

Abstract In this paper, we propose an impulsive chemostat model with microbial competition and nonlinear perturbation. First, thresholds for the extinction of both microoganisms are given. Second, we investigate the persistence in mean and boundedness of the chemostat system by constructing Lyapunov function. Moreover, we obtain the sufficient condition for the existence of an ergodic stationary distribution of the system. At last, numerical simulations are presented, and the results show that the competition between two species tends to make one species disappear from their common habitat, especially when the competition is concentrated in a single resource.

Keywords Impulsive chemostat model, microbial competition, ergodic stationary distribution, extinction, persistence in mean

MSC(2010) 37A50, 37H05, 37N25, 60G10.

1. Introduction

The chemostat is a device used for the continuous cultivation of microorganisms, as shown in Figure 1. It primarily consists of three parts, namely, the feeding device, the cultivation device and the collecting device. The three devices are connected by catheters, and nutrients flow into the culture device at a certain rate for the cultivation of microorganisms in the device. Then, the mixture in the culture device flows into the collection device at the same rate to complete the collection of the culture. As a complex system, it is difficult for scholars to study the natural ecosystem. However, if some minor factors are ignored, the complex system can be simplified to make the influence of research factors more prominent and facilitate the study. The chemostat can only control the velocity and concentration in order to achieve the purpose of simplifying the model.

[†]The corresponding author.
Email address: dydybest@163.com (Y. Dong), mxz721106@sdust.edu.cn (X. Meng)
[1]College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao, Shandong 266590, China
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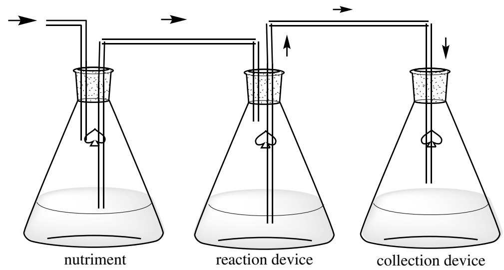

Figure 1. The principle diagram of the chemostat

Many scholars have done a lot of work in chemostat dynamics modeling and analysis, and abundant research results are obtained [3,4,11,14,15,17,19,21]. With the advancement of the research, the models studied by scholars become more complex, and the research results become more realistic. In [13], a deterministic impulsive model which incorporates both toxin input and saturation function is proposed. A stochastic differential equation with nonlinear function was established in [8]. Considering the influence of uncertain factors in the ecological environment [1, 7, 10, 20, 22], Lv, Meng and Wang [12] added random disturbances to the above model and proposed a corresponding stochastic chemostat model. There are complex relationships among microorganisms such as symbiosis, antagonism, predation, competition and parasitism. Among them, microbial competition is to compete for limited space or nutrients for growth, which drives development and evolution [2, 5, 16, 18]. For example, micellar bacteria compete with filamentous bacteria, resulting in the inhibition of growth on both sides. Therefore, based on the existing studies, and considering the microorganisms exposed to toxic environments, the stochastic impulsive chemostat model of competition between two microorganisms is considered as follows.

$$\begin{cases} \left.\begin{aligned} \mathrm{d}S(t) =& \left[D(S_0 - S(t)) - \frac{\mu_1 S(t) x_1(t)}{\delta_1(a_1 + x_1(t))} - \frac{\mu_2 S(t) x_2(t)}{\delta_2(a_2 + x_2(t))}\right]\mathrm{d}t \\ &+ S(t)\left(\sigma_{11} + \sigma_{12} S(t)\right)\mathrm{d}B_1(t), \\ \mathrm{d}x_1(t) =& \left[\frac{\mu_1 S(t) x_1(t)}{a_1 + x_1(t)} - D x_1(t) - r_1 C_0(t) x_1(t)\right]\mathrm{d}t \\ &+ x_1(t)\left(\sigma_{21} + \sigma_{22} x_1(t)\right)\mathrm{d}B_2(t), \\ \mathrm{d}x_2(t) =& \left[\frac{\mu_2 S(t) x_2(t)}{a_2 + x_2(t)} - D x_2(t) - r_2 C_0(t) x_2(t)\right]\mathrm{d}t \\ &+ x_2(t)\left(\sigma_{31} + \sigma_{32} x_2(t)\right)\mathrm{d}B_3(t), \\ \mathrm{d}C_0(t) =& \left[k C_e(t) - g C_0(t) - m C_0(t)\right]\mathrm{d}t, \\ \mathrm{d}C_e(t) =& -h C_e(t)\mathrm{d}t, \end{aligned}\right\} \quad t \neq n\tau, \\ \Delta S(t) = 0, \Delta x_1(t) = 0, \Delta x_2(t) = 0, \Delta C_0(t) = 0, \Delta C_e(t) = u, t = n\tau, n \in Z^+, \end{cases} \tag{1.1}$$

where $B_i(t)$ are independent standard Brownian motions defined on the complete probability space Ω with $B_i(0) = 0$ $(i = 1, 2, 3)$, and $\sigma_{i,j} > 0$ $(i, j = 1, 2, 3)$ represent for the intensities of the white noises on the $S(t)$, $x_1(t)$ and $x_2(t)$ respectively. Besides, the other parameters are defined in Table 1.

This paper is organized as follows. In Section 2, we provide the relevant prelim-