Commun. Comput. Phys. Vol. **34**, No. 2, pp. 530-562
doi: 10.4208/cicp.OA-2022-0104 August 2023

Numerical Solutions of the Electromagnetic Scattering by Overfilled Cavities with Inhomogeneous Anisotropic Media

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Received 2 April 2022; Accepted (in revised version) 22 October 2022

Abstract. In this paper, the electromagnetic scattering from overfilled cavities with inhomogeneous anisotropic media is investigated. To solve the scattering problem, a Petrov-Galerkin finite element interface method on non-body-fitted grids is presented. We reduce the infinite domain of scattering to a bounded domain problem by introducing a transparent boundary condition. The level set function is used to capture complex boundary and interface geometry that is not aligned with the mesh. Non-body-fitted grids allow us to save computational costs during mesh generation and significantly reduce the amount of computer memory required. The solution is built by connecting two linear polynomials across the interfaces to satisfy the jump conditions. The proposed method can handle matrix coefficients produced by permittivity and permeability tensors of anisotropic media. The final linear system is sparse, making it more suitable for most iterative methods. Numerical experiments show that the proposed method has good convergence and realizability. Meanwhile, we discover that the absorbing properties of anisotropic media clearly and positively influence the reduction of radar cross section. It has also been demonstrated that the method can achieve second-order accuracy.

AMS subject classifications: 35Q61, 78A45, 78M30

Key words: Electromagnetic scattering, overfilled cavity, inhomogeneous anisotropic media, non-body-fitted grids, Petrov-Galerkin finite element interface method.

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1 Introduction

Cavity scattering is a topic of great interest in electromagnetism, computational mathematics, and military research. It is a complicated boundary value problem that is commonly used in radar detection. Because a target's radar cross-section (RCS) is one of the most fundamental parameters in radar detection, stealth, and anti-stealth techniques, it is critical for calculating the radar cross-section accurately.

Many studies of cavity scattering assume that the aperture does not protrude beyond the ground plane [1–10], which simplifies the calculation but limits its application. There has been a lot of research interest in overfilled cavities scattering in recent years. The improvised explosive device (IED), for example, is one of the most serious threats to the military on the battlefield. The ability to detect and remove IEDs is improved by investigating the scattering from cavities filled with overfilled media.

The scattering problem by overfilled cavities has received attention from engineering and mathematical communities due to its wide range of applications. Keller and Givoli [11] used the Dirichlet-to-Neumann (DtN) operator to derive boundary conditions on a circle or sphere in engineering. In [12–14], the transparent boundary condition was considered to further reduce the computational domain. Alacikia and Ramahi [15] developed a hybrid finite element boundary integral algorithm to solve the overfilled cavities scattering, and the surface integral equation was used to truncate the protruding portions. Huang and Wood [16] reviewed the mathematical study of the scattering by overfilled cavities and proposed an algorithm based on a variational formulation to analyze scattering from two-dimensional overfilled cavities in an infinite plane. On this basis, Wood [17] established the existence and uniqueness of internal problems, using the DtN operator on the artificial boundary. Du [18] presented a new transparent boundary condition on a semi-ellipse using the Mathieu function for scattering from overfilled cavities with homogeneous media, which reduced the computational domain. Furthermore, some theoretical discussions on time-domain scattering from an overfilled cavity can be found in [19–22].

However, most of the existing literature on scattering from overfilled cavities restricts the study to cavity scattering with homogeneous media. In practice, because of their many special physical properties, inhomogeneous anisotropic media have a wide range of applications in microwave circuits, microstrip antennas, and radiation elements. The use of layered anisotropic media in the radiation device, for example, improves antenna efficiency. Due to the difficulty of capturing the complex geometry of the interface and the convergence of the method for solving the inhomogeneous anisotropic problem with tensor coefficients, electromagnetic scattering from protruded cavities filled with inhomogeneous anisotropic media becomes more difficult.

Some previous work has been done on elliptic interface problems. The immersed interface method [23,24] produces a high-order compact scheme, which represents a significant advancement in the solution of interface problems. The immersed finite element method [25–28], which is based on structured meshes independent of the interface,