Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **15**, No. 6, pp. 1602-1630

DOI: 10.4208/aamm.OA-2022-0006
December 2023

High Order Deep Domain Decomposition Method for Solving High Frequency Interface Problems

Zhipeng Chang[1,2], Ke Li[3], Xiufen Zou[1,*] and Xueshuang Xiang[2,*]

[1] *School of Mathematics and Statistics, Wuhan University, Wuhan, Hubei 430072, China*
[2] *Qian Xuesen Laboratory of Space Technology, China Academy of Space Technology, Beijing 100875, China*
[3] *Information Engineering University, Zhengzhou, Henan 450001, China*

Received 9 January 2022; Accepted (in revised version) 20 October 2022

Abstract. This paper proposes a high order deep domain decomposition method (HOrderDeepDDM) for solving high-frequency interface problems, which combines high order deep neural network (HOrderDNN) with domain decomposition method (DDM). The main idea of HOrderDeepDDM is to divide the computational domain into some sub-domains by DDM, and apply HOrderDNNs to solve the high-frequency problem on each sub-domain. Besides, we consider an adaptive learning rate annealing method to balance the errors inside the sub-domains, on the interface and the boundary during the optimization process. The performance of HOrderDeepDDM is evaluated on high-frequency elliptic and Helmholtz interface problems. The results indicate that: HOrderDeepDDM inherits the ability of DeepDDM to handle discontinuous interface problems and the power of HOrderDNN to approximate high-frequency problems. In detail, HOrderDeepDDMs ($p>1$) could capture the high-frequency information very well. When compared to the deep domain decomposition method (DeepDDM), HOrderDeepDDMs ($p>1$) converge faster and achieve much smaller relative errors with the same number of trainable parameters. For example, when solving the high-frequency interface elliptic problems in Section 3.3.1, the minimum relative errors obtained by HOrderDeepDDMs ($p=9$) are one order of magnitude smaller than that obtained by DeepDDMs when the number of the parameters keeps the same, as shown in Fig. 4.

AMS subject classifications: 35Q68, 65N55, 68T99

Key words: Deep neural network, high order methods, high-frequency interface problems, domain decomposition method.

---

*Corresponding author.
Emails: xfzou@whu.edu.cn (X. Zou), xiangxueshuang@qxslab.cn (X. Xiang)

1 Introduction

High-frequency interface problems commonly appear in various scientific and engineering applications, such as the high-frequency scalar wave equation and Schrödinger equation with a sharp interface [15]. The traditional numerical methods, such as finite element methods (FEMs) and finite difference methods (FDMs), cannot effectively predict the behavior of the high-frequency waves due to the discontinuities caused by the interface. To solve the interface problems, [17] proposes a domain decomposition method (DDM), named "Schwarz alternating method", which transforms a discontinuous interface problem into some continuous sub-problems and then solves them separately. After the parallel computing capability becomes available, the Schwarz alternating method is further developed, and many efficient methods are proposed, such as the parallel Schwarz method [14], the multiplicative Schwarz method [1,18], and the additive Schwarz method [4]. Traditional numerical methods such as FEMs and FDMs are usually applied for sub-problems in these domain decomposition methods. However, they are mesh dependent, and mesh generation is expensive, especially for complex PDEs, e.g., those with complex interfaces or boundaries. In addition, the highly oscillatory nature of the high-frequency problems also brings significant challenges for finding numerical solutions of high accuracy.

Recently, deep-learning-based numerical methods for solving partial differential equations(PDEs) have received much attention due to their meshless advantage. They are also used in combination with DDM to solve PDEs on complex domains to pursuit greater accuracy and efficiency. One popular way is to replace the sub-domain solvers with deep-learning-based solvers such as physics-informed neural network(PINN) [16] and deep ritz method [22] in the classical overlapping Schwarz approach. Under this framework, [13] proposes a DeepDDM, which leverages PINN to discretize the sub-problems divided by DDM and exchanges the sub-problem information across the interface by adjusting the boundary term in the solution of each sub-problem. [10] proposes a cPINN, where the computational domain is also divided and the flux continuity in the strong form is enforced along the sub-domain interfaces. [9] further proposes a more generalized space-time domain decomposition approach. In fact, the idea of domain decomposition is also used to solve interface problems, where different neural networks are often employed for different sub-domains divided by the interface to deal with the dramatical change across the interface of the solution, and these neural networks are weakly coupled by the interface conditions [6, 7]. Besides, other ways of dealing with discontinuous interfaces also exist. For example, the d-dimensional piecewise continuous solution of the interface problem can be continuously extended in $(d+1)$-dimensional space by augmenting a variable, so that the continuous augmented function can be approximated by a shallow neural network efficiently [8,12]. Although these methods have been successful for solving interface problems to a certain extent, challenges still exist when they are applied to high-frequency interface problems. For example, these conventional neural networks preferentially approximate the low-frequency components of the target